

A-level Physics Tutor Guides

A-level Physics  
COURSE NOTES

**WAVES**

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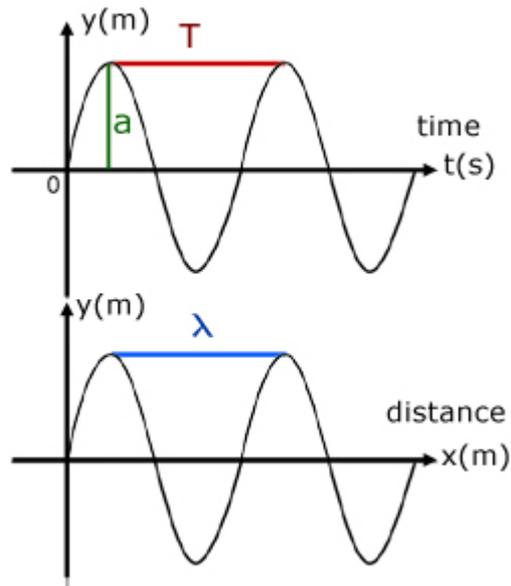
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## Properties of Waves

### definitions



**Wavelength (  $\lambda$  )** is the horizontal distance along a wave between similar particles of the wave.

**Displacement** is the distance of a particle of the wave from its equilibrium position at any particular time

**Amplitude (  $a$  )** is the maximum displacement of a particle of the wave from its equilibrium position.

**Period (  $T$  )** is the time for one complete oscillation of the wave.

**Frequency (  $f$  )** is the number of waves produced per second.

**Velocity (  $v$  )** the velocity of a particle of a wave in the direction the wave is travelling.

Units		
wavelength	metre	m
displacement	metre	m
period	second	s
frequency	Hertz (Hz)	$s^{-1}$
velocity	metres per second	$ms^{-1}$

### Equations

The velocity ( $v$ ) of a wave is expressed in terms of its frequency ( $f$ ) and wavelength ( $\lambda$ ):

$$v = f\lambda$$

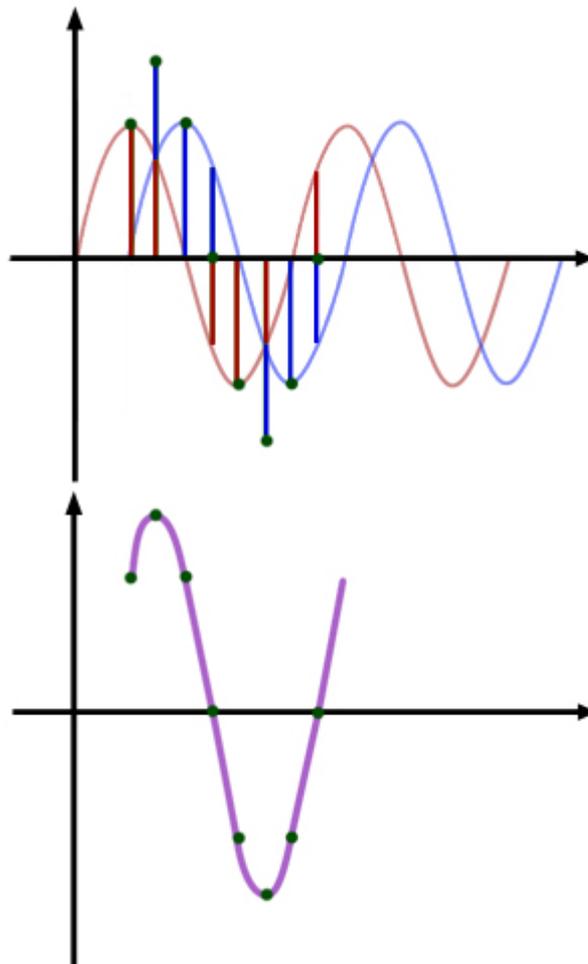
The period and frequency of a wave are the inverse of each other:

$$T = \frac{1}{f} \quad f = \frac{1}{T}$$

### Superposition

Superposition is when two waves are superimposed on each other and add up. The phenomenon is described by the Principle of Superposition, which states:

**When two waves are travelling in the same direction and speed, at any point on the combined wave the total displacement of any particle equals the vector sum of displacements of the waves.**



Look carefully at the diagram. The blue and red displacements add up algebraically.

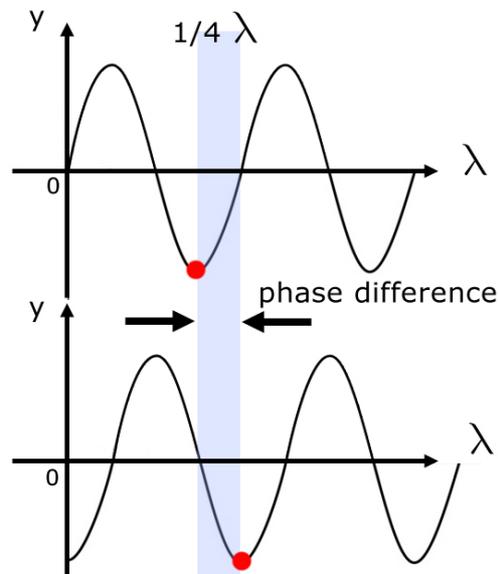
Above the line is positive. Below the line is negative.

Hence a red displacement, above the line, on top of a blue displacement (of equal magnitude) below the line, will cancel out. This produces a point on the horizontal axis.

A red displacement, above the line, on top of a blue displacement, also above the line, will produce a displacement above the line equal to their sum.

A red displacement, below the line, below a blue displacement, also below the line, will produce a displacement below the line equal to their sum.

### phase difference



The phase difference of two waves is the horizontal distance a similar part of one wave leads or lags the other wave.

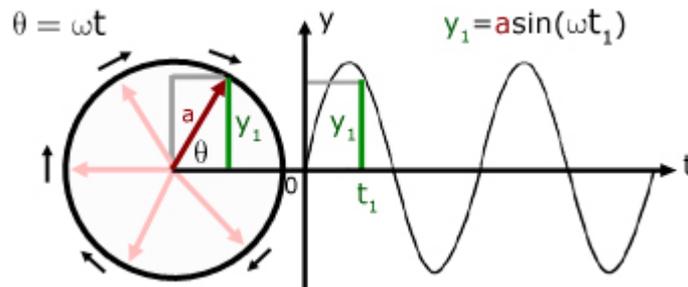
Phase difference is measured in fractions of a wavelength, degrees or radians.

$$\text{fraction of } \lambda = \text{fraction} \times 360^\circ = \text{fraction} \times 2\pi$$

In the diagram (above), the phase difference is  $\frac{1}{4} \lambda$ .

This translates to  $90^\circ$  ( $\frac{1}{4}$  of  $360^\circ$ ) or  $\frac{\pi}{2}$  ( $\frac{1}{4}$  of  $2\pi$ ).

equation of a sinusoidal wave



The graph represents a sinusoidal wave with displacement  $y$  at time  $t$ , vibrating at a frequency  $f$  and amplitude  $a$ . The motion can be described by the equation:

$$y = a \sin(2\pi f t)$$

We can understand how this equation is constructed by introducing  $\omega$  (omega), the angular velocity (units  $\text{rad s}^{-1}$ ).

$$T = \frac{1}{f} \qquad T = \frac{2\pi}{\omega}$$

$$\frac{1}{f} = \frac{2\pi}{\omega} \qquad \underline{\omega = 2\pi f}$$

Substituting for  $2\pi f$ , our equation then becomes,

$$y = a \sin(\omega t)$$

The diagram shows how the value of the function ( $y$ ) is calculated from the radius ( $a$  - red) of the circle sweeping out an angle  $\theta$  (theta).

angle swept out = angular velocity x time of sweep

$$\theta = \omega t$$

From simple trigonometry, the value of  $y_1$  (green) is equal to  $a \sin \theta$ .

The angle swept out at time  $t_1$  is  $\omega t_1$  where  $\omega$  is the angular velocity. This is a measure of the rotation of the  $a$ -vector in radians per second.

So the value  $y_1$  at time  $t_1$  is given by:

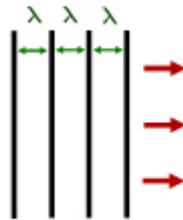
$$y_1 = a \sin(\omega t_1)$$

# Huygens' Construction

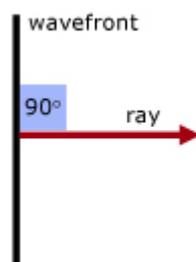
## Introduction

To understand Huygens' Construction it is important to know the relation between **wavefronts** and **rays**.

A **wavefront** represents the leading edge of one complete wave. So the perpendicular distance between two wavefronts represents one wavelength.



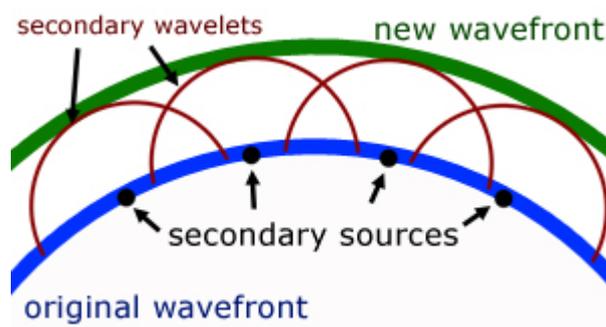
A **ray** is simply the direction of travel of a wavefront. Hence wavefronts and rays are always at right angles to each other.



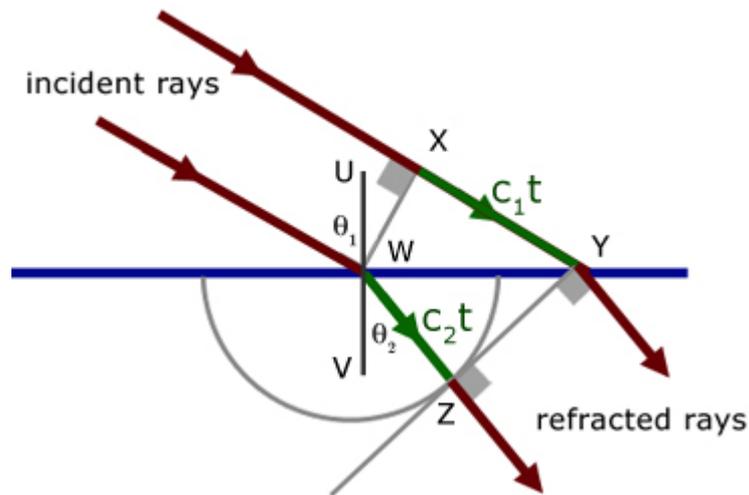
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## Huygens' postulate

Huygens postulated that points on the wavefronts themselves were the source of small waves and that they combined to produce further wavefronts.



## Refraction



Consider the wavefront **WX** at right angles to two parallel incident rays.

$c_1$  is the velocity of light in the medium above the blue line

$c_2$  is the velocity of light in the medium below the blue line

Let **t** be the time interval for the wavefront to travel from **X** to **Y**.

When the wavefront **WX** enters the medium (below the blue line) a wavelet of radius  $c_2t$  is produced.

The new wavefront is found by drawing a tangent to the circle from **Y**.

If the tangent meets the circle at point **Z**, then the new wavefront is **YZ**.

Using some simple geometry, an important relationship can be found relating the angle of incidence  $\theta_1$ , the angle of refraction  $\theta_2$  with the velocity of light  $c_1$  and  $c_2$  in the respective media.

Looking at the diagram, around point **W**,

$$\angle YWX + \angle UWX = 90^\circ$$

$$\theta_1 + \angle UWX = 90^\circ$$

$$\therefore \underline{\theta_1 = \angle YWX}$$

in triangle **WYZ** ,

$$\angle WYZ + \angle YWZ = 90^\circ$$

$$\theta_2 + \angle YWZ = 90^\circ$$

$$\therefore \underline{\theta_2 = \angle WYZ}$$

Looking at triangles **WXY** and **WYZ** ,

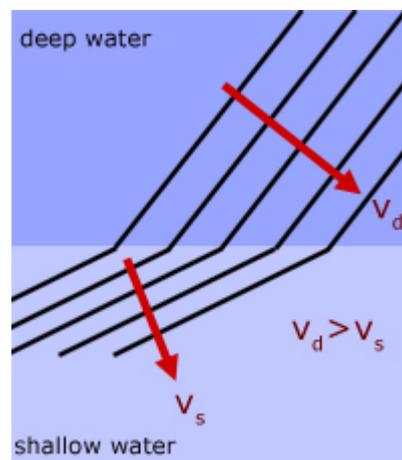
$$\frac{\sin(\angle YWX)}{\sin(\angle WYZ)} = \frac{\left(\frac{WX}{YW}\right)}{\left(\frac{WZ}{YW}\right)} = \frac{WX}{WZ} = \frac{c_1 t}{c_2 t} = \frac{c_1}{c_2}$$

substituting for angles **YWX** and **WYZ** from above, this becomes:

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{c_1}{c_2} = \mu$$

where  $\mu$  (mu) is the refractive index between the two media.

The effect can be observed in a **ripple tank**. Water waves travel faster in deeper water than in shallow.



Note the change in wavelength. Wavelength is shorter in shallower water. This is a result of the frequency of the waves being the same, while the wavelength and the velocity can change.

$$v = f\lambda$$

where,

$v$  is the velocity of the wave  
 $f$  is the wave frequency  
 $\lambda$  is the wavelength

rearranging to make the frequency  $f$  the subject,

$$f = \frac{v}{\lambda}$$

$$f = \frac{v_s}{\lambda_s} \quad f = \frac{v_d}{\lambda_d}$$

$$\frac{v_s}{\lambda_s} = \frac{v_d}{\lambda_d}$$

$$\lambda_d v_s = \lambda_s v_d$$

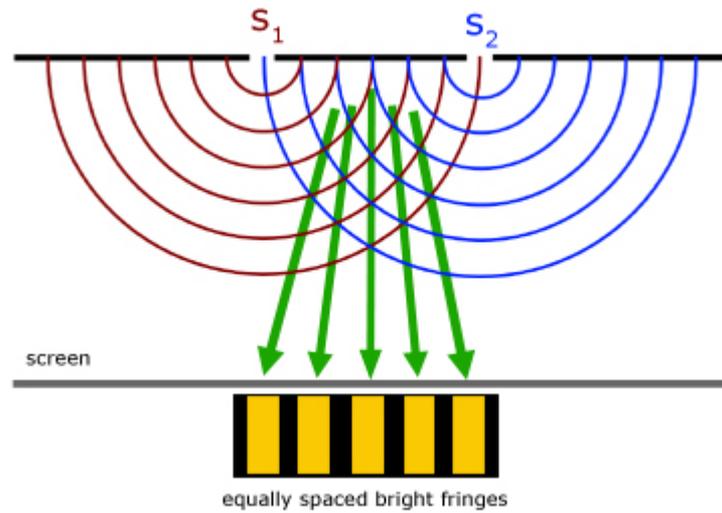
$$\frac{\lambda_d}{\lambda_s} = \frac{v_d}{v_s}$$

So the ratio of the wavelengths equals the ratio of the velocities.

A high velocity gives a large wavelength.

A small wavelength indicates a low velocity.

### Interference - Young's Fringes



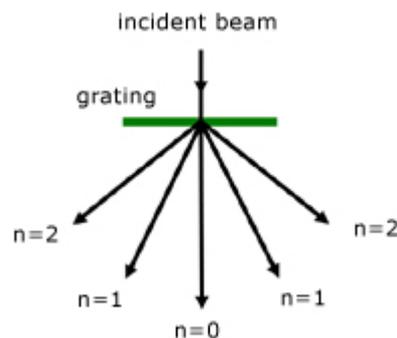
$S_1$  and  $S_2$  act as sources of waves in phase with each other.

**Bright fringes** (yellow) are produced in the directions where light interferes constructively.

On the diagram, this corresponds to where the red and blue lines cross.

### Diffraction Grating

A diffraction grating is a rectangular piece of thin glass with many equally spaced parallel lines ruled on it. A typical grating will have about 600 lines drawn per millimetre.



When a parallel beam of monochromatic light is incident normally to a grating, distinct images (maxima) are produced at specific angles.

The effect is described by the equation:

$$d \sin \theta = n \lambda$$

where

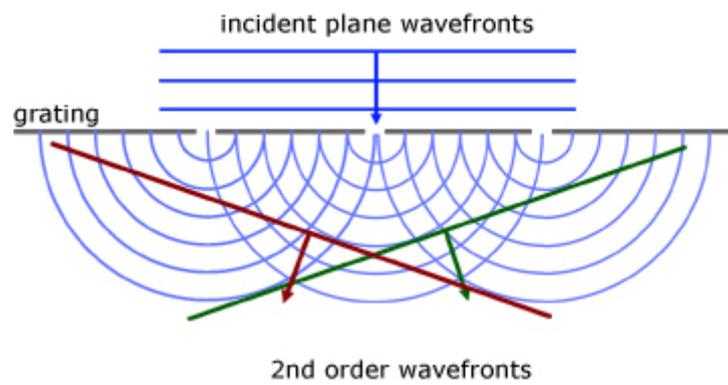
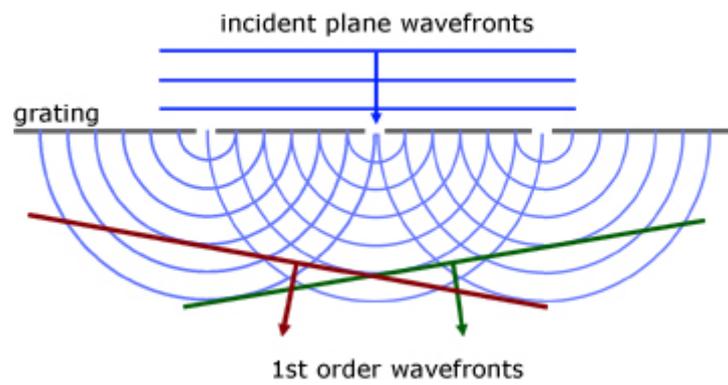
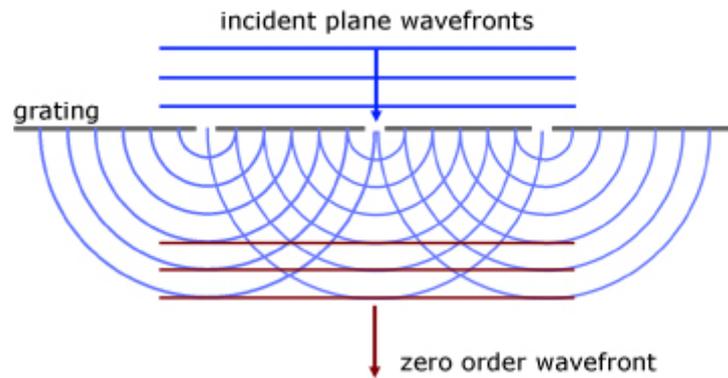
$d$  is the distance between lines

$n$  is the order of the maxima (0, 1, 2 ... )

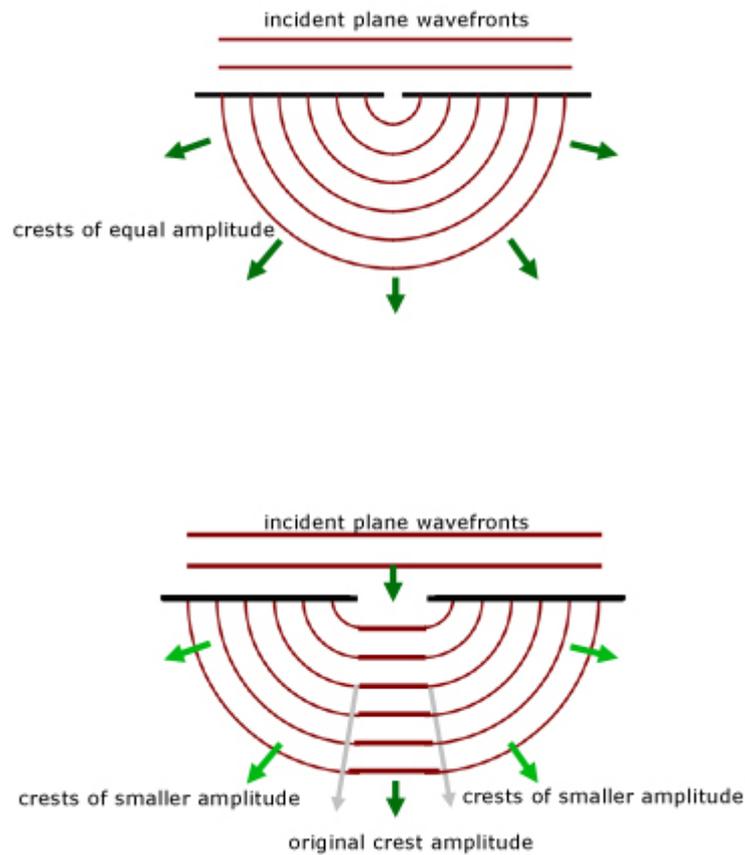
$\theta$  is the angle between the maxima beam and the incident beam

$\lambda$  is the wavelength of the light

The following sequence of diagrams show how plane wavefronts produce discrete maxima at specific angles.



Water waves at a gap



As the gap enlarges, the two edges act as centres for circular waves.

It is observed that the circular waves have smaller crests than the plane waves passing through the middle.

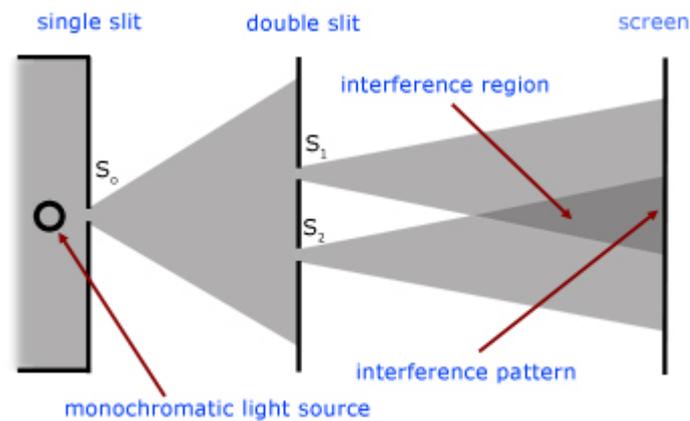
Notice that the lengths of the wavefronts passing through the gap get wider. The grey arrows indicate the change. This demonstrates **diffraction at an edge**.

## Interference of Light

### Conditions for interference

1. The waves from light sources must be **coherent** with each other. This means that they must be of the same frequency, with a constant phase difference between them.
2. The amplitude (maximum displacement) of interfering waves must have the same magnitude. Slight variations produce lack of contrast in the interference pattern.

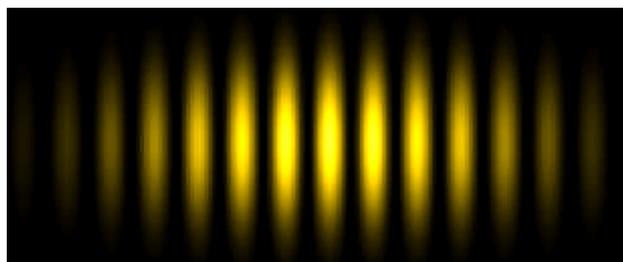
### Young's Double Slit Experiment - Apparatus



It is important to realise that the diagram is **not** to scale.

Typically the distance (**D**) between the double slits and the screen is  $\sim 0.2$  m (20 cm).  
 The distance (**a**) between the double slits is  $\sim 10^{-3}$ m (1mm).  
 The preferred monochromatic light source is a sodium lamp.

### Young's Double Slit Experiment - Display



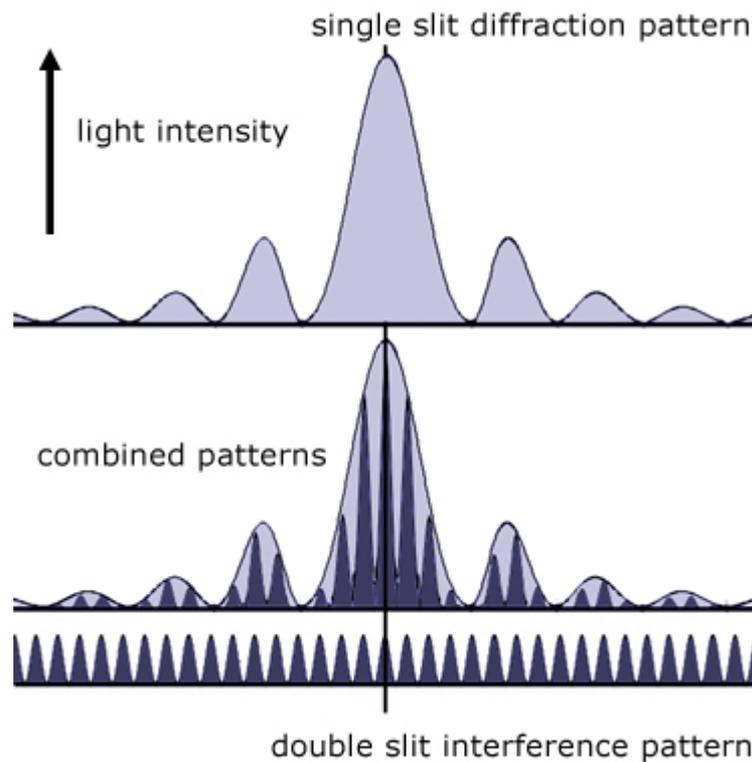
You will notice some dimming in the image from the centre travelling outwards. This is because the regular light-dark bands are superimposed on the light pattern from the single slit.

The intensity pattern is in effect a combination of both the single-slit diffraction pattern and the double slit interference pattern.

The amplitude of the diffraction pattern **modulates** the interference pattern.

In other words, the diffraction pattern acts like an envelope containing the interference pattern.

The image above is taken from the **central maximum** area of a display.



#### Young's Double Slit Experiment - theory

The separation ( $y$ ) of bright/dark fringes can be calculated using simple trigonometry and algebra.

Consider two bright fringes at **C** and **D**.

For the fringe at **C**, the method is to find the **path difference** between the two rays **S<sub>1</sub>C** and **S<sub>2</sub>C**. This is then equated to an exact number of wavelengths **n**.

A similar expression is found for the fringe at **D**, but for the number of wavelengths **n+1**.

The two expressions are then combined to exclude **n**.



Using 'the difference of two squares' to expand the LHS,

$$(S_2C)^2 - (S_1C)^2 = (S_2C - S_1C)(S_2C + S_1C)$$

hence,

$$(S_2C - S_1C)(S_2C + S_1C) = 2ax_C$$

The path difference  $S_2C - S_1C$  is therefore given by:

$$(S_2C - S_1C) = \frac{2ax_C}{(S_2C + S_1C)}$$

In reality,  $a \sim 10^{-3}\text{m}$  and  $D \sim 0.2\text{ m}$ . The length  $a$  is much smaller than  $D$ . The two rays  $S_2C$  and  $S_1C$  are roughly horizontal and each equal to  $D$ ,

so,

$$(S_2C - S_1C) = \frac{2ax_C}{2D}$$

cancelling the 2's,

$$(S_2C - S_1C) = \frac{ax_C}{D}$$

For a bright fringe at point  $C$  the path difference  $S_2C - S_1C$  must be a whole number ( $n$ ) of wavelengths ( $\lambda$ ).

Hence,

$$n\lambda = \frac{ax_C}{D}$$

Rearranging to make  $x_C$  the subject,

$$x_C = \frac{n\lambda D}{a}$$

Similarly for the next bright fringe at  $D$ , when the path difference is one wavelength longer ( $n+1$ ),

$$x_D = \frac{(n+1)\lambda D}{a}$$

hence the fringe separation  $x_D - x_C$  is given by,

$$\begin{aligned} x_D - x_C &= \frac{(n+1)\lambda D}{a} - \frac{n\lambda D}{a} \\ &= \frac{n\lambda D}{a} + \frac{\lambda D}{a} - \frac{n\lambda D}{a} \\ &= \frac{\lambda D}{a} \end{aligned}$$

assigning the fringe separation the letter  $y$ ,

$$y = \frac{\lambda D}{a}$$

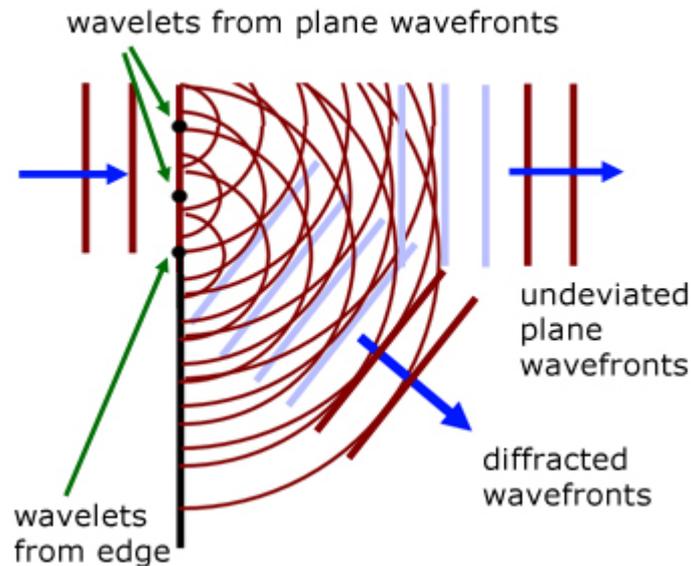
or with wavelength  $\lambda$  the subject,

$$\lambda = \frac{ay}{D}$$

## Diffraction of Light

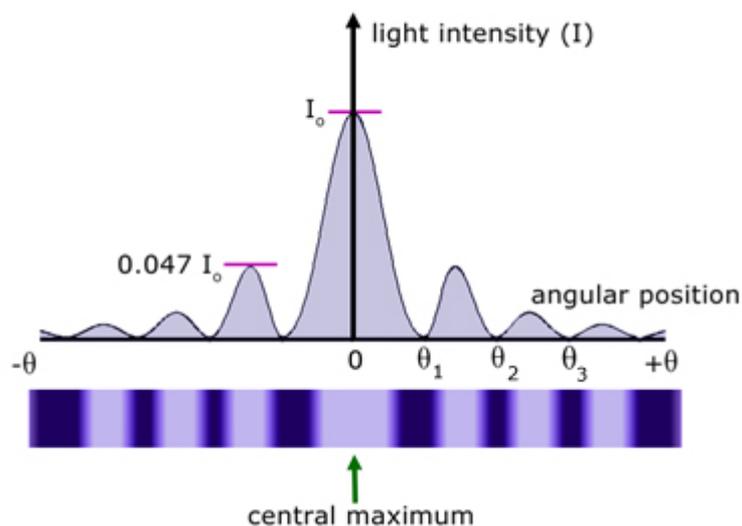
### Basic concepts

Diffraction is the bending of light at an edge as a result of the superposition of wavelets from a plane wavefront.



### Single Slit

The diffraction pattern is graphed in terms of intensity and angle of deviation from the central position.



Note that the central maximum is **twice the width** of other maxima and that all these have the **same width**.  
The diagram and image give a false impression regarding the relative brightness of fringes.

The secondary maxima are considerably dimmer than the central maximum (4.7% of the brightness).

To frame an equation that predicts outcomes from this waveform, we must re-visit work on the **superposition** of waves.

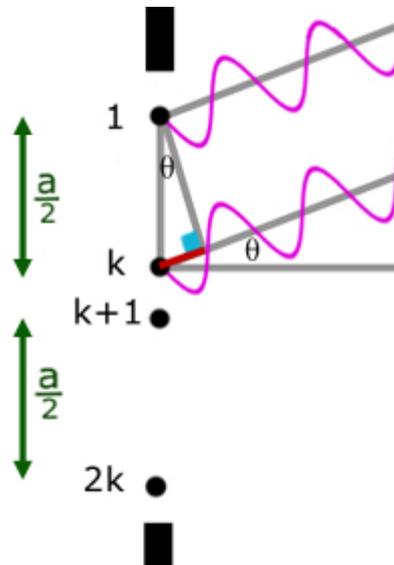
The dark fringes (minima) are where pairs of light waves are in anti-phase and cancel out.

Consider pairs of light waves interfering with each other across the width of the slit. The first light wave (1) in the upper half of the slit interferes destructively with the light wave (k) from the middle of the slit.

Then,

light wave 2 interferes with wave k+1,  
light wave 3 interferes with wave k+2,

and so on, until light wave k+1 interferes with light wave 2k. For each pair of light waves the phase difference is half a wave length and the vertical distance between wave points is  $a/2$ .



So, for the **first minimum**, we look at the first and last waves in this segment ( $a/2$ ) with a phase difference of  $\lambda/2$  ( $\pi$  - pi radians).

Using simple trigonometry, the path difference (in red) is equal to  $a/2 \sin(\theta)$ .

Equating,

$$\frac{\lambda}{2} = \frac{a}{2} \sin(\theta)$$

$$\lambda = a \sin(\theta)$$

Unfortunately lack of space precludes the derivation of further minima.

As a general rule, the angular positions  $\theta$  of the minima are given by:

$$n\lambda = a \sin(\theta)$$

In practice the slit width (**a**) is much larger than the wavelength ( **$\lambda$** ) of light used.  
 **$a \gg \lambda$**

Rearranging our single slit equation, with  $n=1$  for the first minimum,

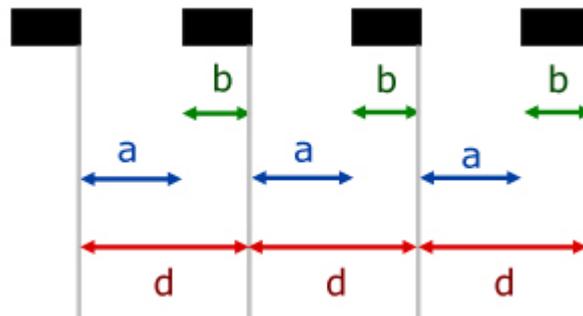
$$\sin(\theta) = \frac{\lambda}{a}$$

As a result of the ratio  $\lambda/a$  being very small,  **$\sin(\theta) \approx \theta$**  in value.

So the equation becomes:

$$\theta = \frac{\lambda}{a}$$

### Multiple slit diffraction



**N** - the no. of clear slits drawn on an opaque glass slide

**a** - the width of a clear area

**b** - the width of an opaque area

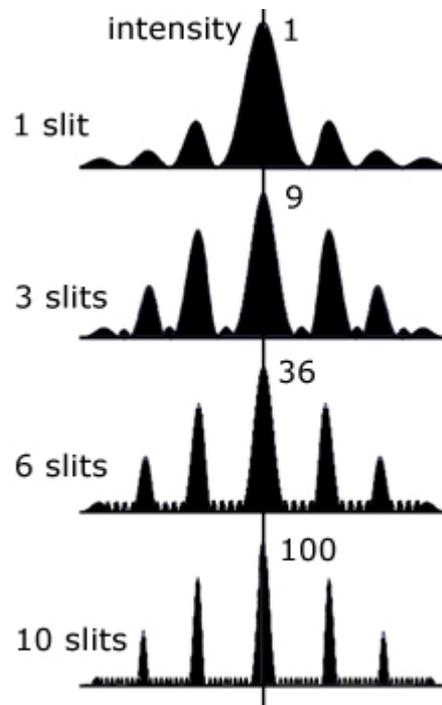
**d** - width of a clear and an opaque area

**L** - length of glass slide

$$d = a + b$$

$$N = L/d$$

The diffraction patterns below are obtained by varying **N**.



Note that with increasing N,

1. as the intensities of the principal maxima increase the intensities of the subsidiary maxima decrease
2. the sharpness of the principal maxima increases
3. the angular position of maxima remains the same
4. the absolute intensity of maxima increases

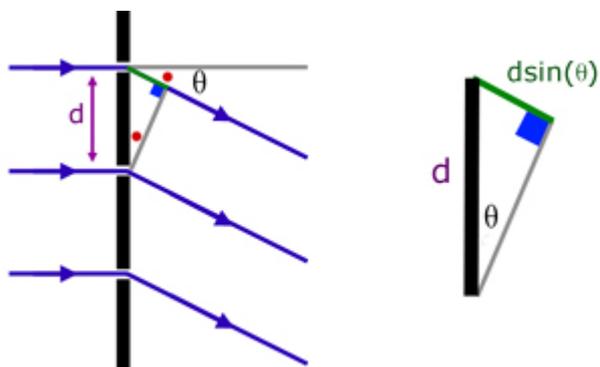
For a particular value of N, the effect of altering the sum  $a+b$  is to:

1. alter the angular positions of the principal maxima
2. alter the relative intensities of principal maxima

### Diffraction grating

The clear spaces on a diffraction grating act as equally spaced slits.

So light rays diffracted at the same angle ( $\theta$ ) and in phase with each other will interfere constructively. Whenever this happens a bright fringe called a principal maxima is produced.



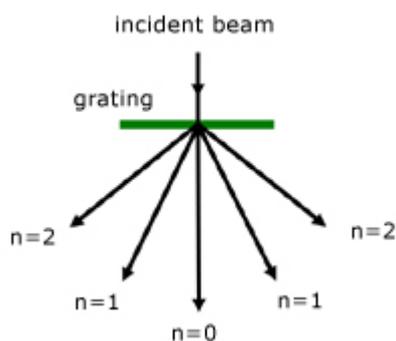
The path difference between successive light rays must therefore be a whole number ( $n$ ) of wavelengths ( $\lambda$ ).

Using simple trigonometry, if  $d$  is the distance between slits, then the path difference is  $d \sin(\theta)$ .

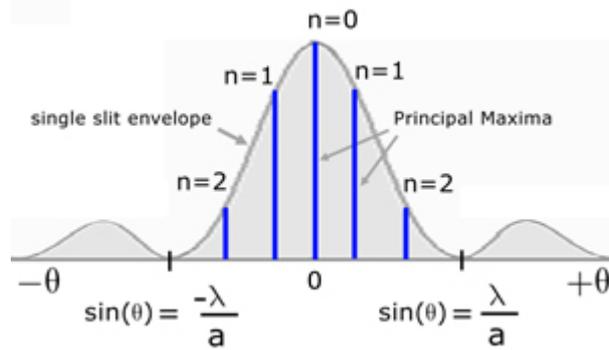
Hence,

$$d \sin(\theta) = n \lambda \quad (n = 0, 1, 2, 3 \dots)$$

So discrete bright fringes (principal maxima) are produced at specific angles for particular wavelengths of light.

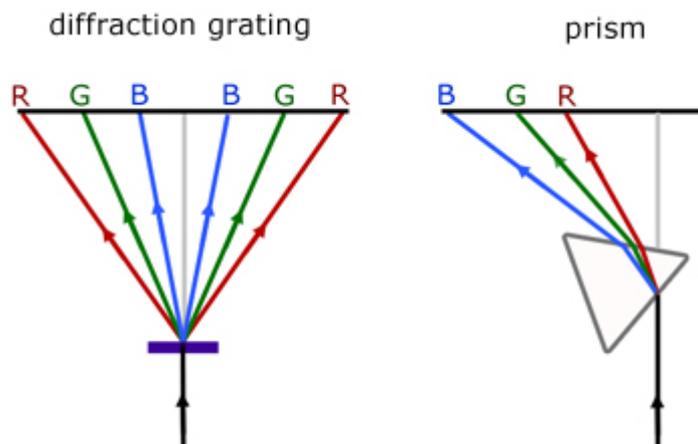


It must be noted that the distribution of light intensity across a diffraction display remains that of a single slit.



### Grating and Prism spectra compared

From intermediate physics you may remember 'red rays refract least', illustrated by the prism diagram below.



However, for diffraction gratings the deviation of coloured rays is the reverse. If we rearrange our diffraction grating equation to make wavelength the subject, making  $n=1$  for the first principal maxima to the right or left.

$$\sin(\theta) = \frac{\lambda}{d}$$

We can see that when wavelength  $\lambda$  is large (eg red light)  $\sin(\theta)$  is large, because  $\lambda$  and  $\sin(\theta)$  are directly proportional to each other. So  $\theta$  is also large. Now consider the case when wavelength  $\lambda$  is small (eg blue light). By proportion,  $\sin(\theta)$  will also be small. So  $\theta$  will be small.

### Grating spectra

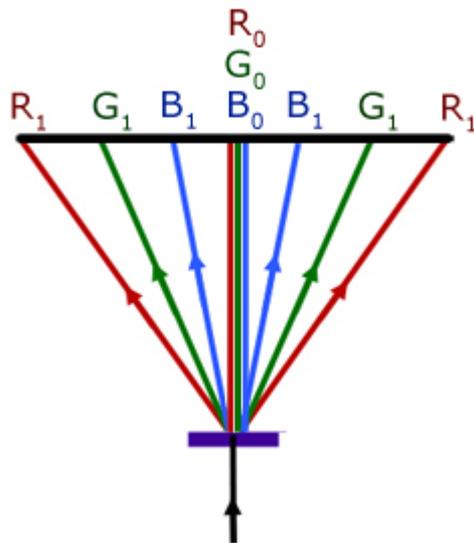
When using a diffraction grating to examine spectra and measure wavelength only the principal maximum with  $n=1$  is used.

The first principal maximum at  $n=0$  is a bright fringe with all wavelengths mixed.

So if white light, made from a mixture of pure red, green and blue wavelengths were examined, the central fringe would be pure white.

Examining the principal maxima on either side, with order  $n=1$  we find different coloured fringes deviated at different angles. Each fringe has a fair degree of brightness.

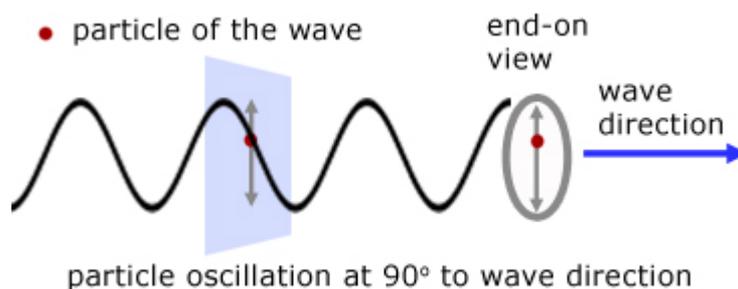
Note that the principal maxima for  $n=2$  have greater deviations (right and left) and are considerably dimmer.



## Polarization of Light

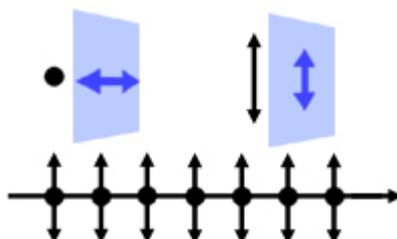
### Explanation

Light waves are **transverse waves**, where 'particles' of the wave oscillate in a line at right angles to the direction of travel.



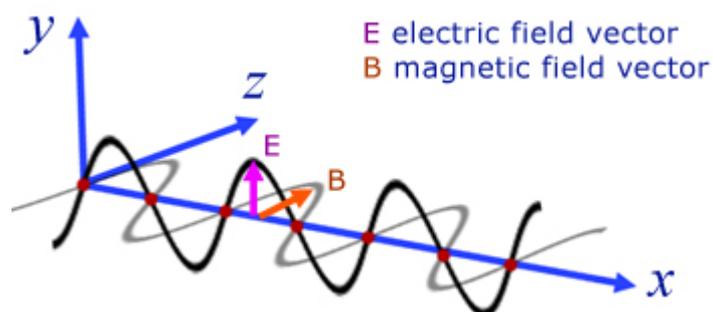
In a light beam there are many waves with lines of oscillation set at random angles. Polarization is the production of waves oscillating in **one** plane (ie with one line of oscillation) from a source of randomly oscillating waves.

In this work it is convenient to represent light waves in a simplified form. Only the vertical and horizontal waves are represented when explaining the various phenomena around polarization.



**note:** A light wave is actually two waves, in phase and oriented at  $90^\circ$  to each other. One wave is E, the electric field vector and the other is B, the magnetic field vector.

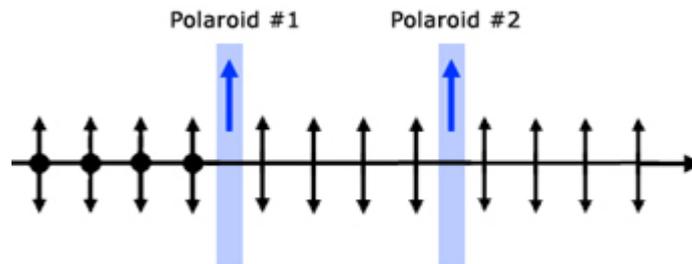
When interacting with matter, the E wave is much more important than the B wave. So for simplicity the B wave is ignored.



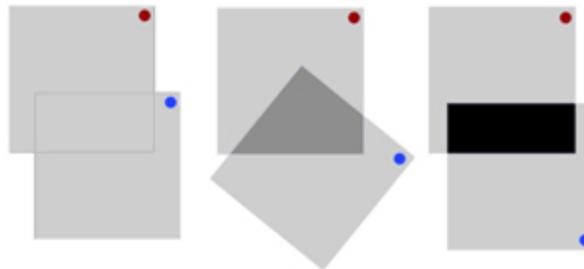
## Polaroid

Polaroid\* is a material, usually plastic, which allows light to pass but only where the wave oscillates in ONE particular orientation. This orientation is called the **reference direction**.

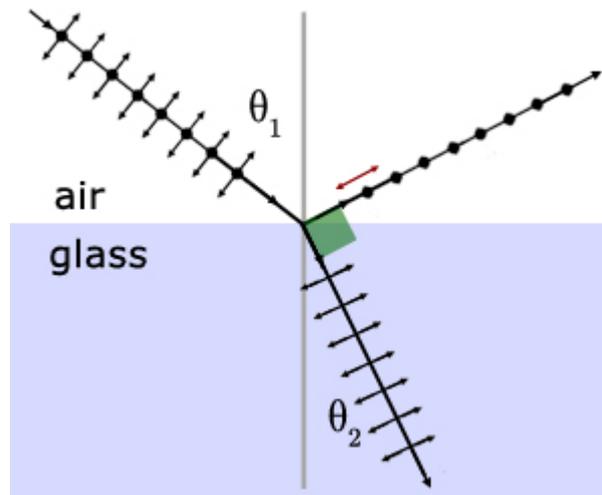
Consider a single piece of Polaroid where the reference direction is vertical. When a beam of unpolarized light is directed at the Polaroid, a beam of vertically polarized light rays is transmitted.



If a second piece of Polaroid similar to the first is placed in front of the polarized beam the beam will be transmitted with only a very small loss in intensity. However, if the second Polaroid is rotated through  $90^\circ$  no light is transmitted. An observer looking in this way at a bright light would see the light reduce in brightness as the Polaroid is rotated until the image disappears completely in darkness.



\***Polaroid** is a trade name. Polaroid is actually a sandwich of sheets of nitrocellulose. The nitrocellulose is seeded with crystals of quinine iodosalphate, which have the property of aligning their axes in one particular direction. This property (**dichroism**) screens out all light except that with the same orientation.

Reflection

$\theta_1$  is the angle of incidence  
 $\theta_2$  is the angle of refraction  
 $\mu$  is the refractive index (air- glass)

Applying Snell's Law,

$$\mu = \frac{\sin(\theta_1)}{\sin(\theta_2)} \quad (i)$$

but  $\theta_1$  and  $\theta_2$  are complementary (they add up to  $90^\circ$ ),

therefore, 
$$\theta_2 = 90^\circ - \theta_1$$

substituting for  $\theta_2$  into equation (i)

$$\mu = \frac{\sin(\theta_1)}{\sin(90^\circ - \theta_1)} \quad (ii)$$

but 
$$\cos(\theta_1) = \sin(90^\circ - \theta_1)$$

substituting for  $\sin(90^\circ - \theta_1)$  into equation (ii)

$$\begin{aligned}\mu &= \frac{\sin(\theta_1)}{\cos(\theta_1)} \\ &= \tan(\theta_1)\end{aligned}$$

$$\theta_1 = \tan^{-1}(\mu)$$

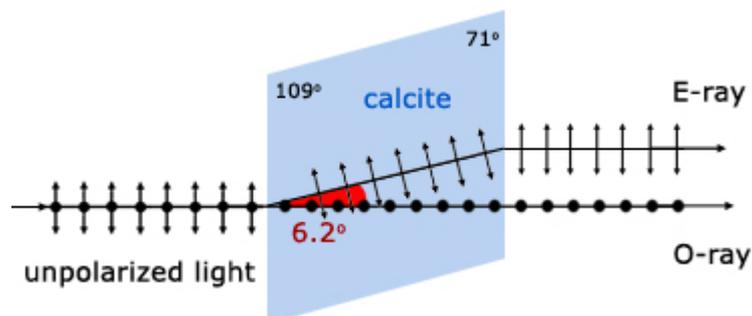
In other words, the angle of incidence has a tangent value equal to the refractive index.

The relationship is called **Brewster's Law**.

The angle of incidence is also called the **polarizing angle** or the **angle of polarization**.

### Double Refraction

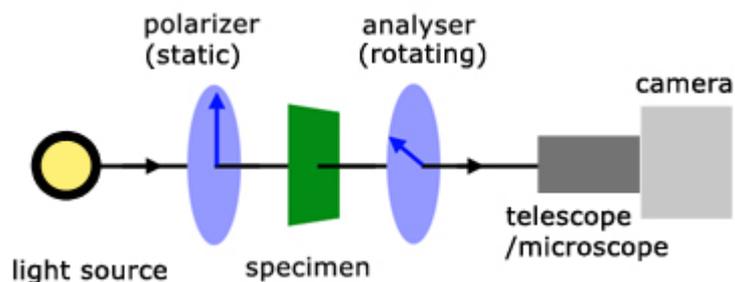
Double refraction is the property particular crystals(eg calcite) have that allows them to split an unpolarized ray into two rays that are plane polarized at right angles to each other.



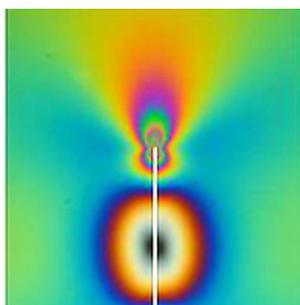
In this particular orientation of the crystal, the **Ordinary (O)** ray has normal incidence, while the **Extraordinary (E)** ray follows a path as if it had an angle of refraction in the calcite of  $6.2^\circ$

### Stress analysis & mineralogy uses

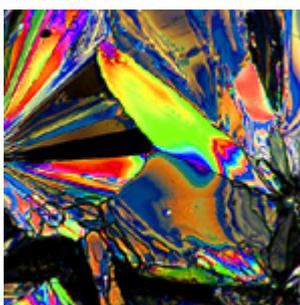
Polarized light for stress analysis and looking at minerals is used in the same manner. In each case the specimen is illuminated with polarized light from one side and then observed through Polaroid from the other.



Stress analysis is important in the design and manufacture of parts for industry. Replicas in clear plastic are put under stress to highlight weaknesses in molded shapes. The images produced illustrate the photo-elastic behaviour of the plastic.



Polarization is also used in mineralogy to identify the crystalline components of different rocks.

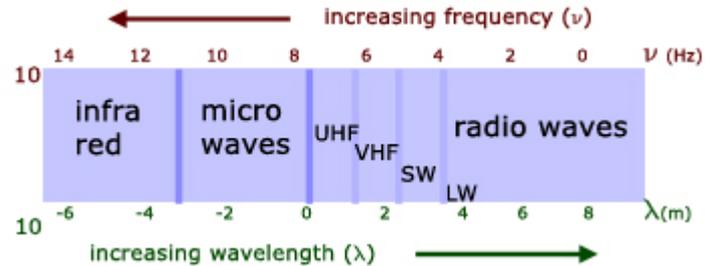
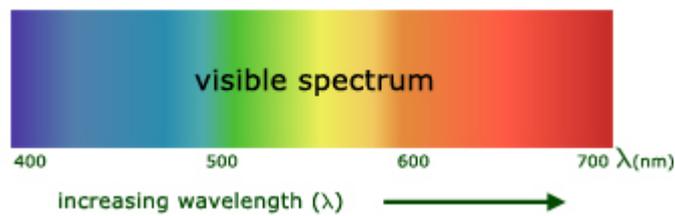
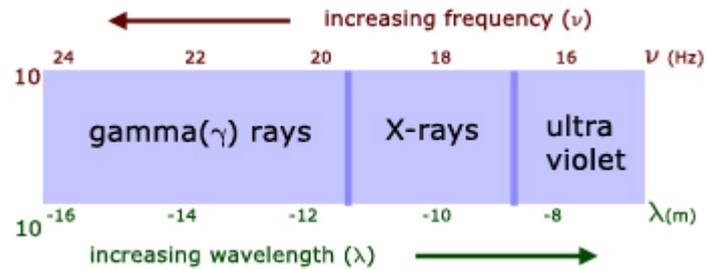


The different colours are a result of **double refraction** or as it is sometimes called, **birefringence**.

The strength of double refraction in crystals dictates the range and depth of colours produced. The colours themselves are a result of interference between rays.

## Electromagnetic Waves

### Electromagnetic Spectrum



### **$\gamma$ (gamma)-rays**

#### generation :

1. nuclear fission
2. nuclear fusion
3. radioactive decay
4. elementary particle interactions

#### properties :

1. very penetrating
2. produce weak ionisation
3. produce weak fluorescence
4. affect photo film

detection :

1. Geiger-Muller tube
2. scintillation counter
3. solid state detectors
4. photo film

**X-rays**generation :

1. electron deceleration
2. electron energy level changes in atoms

properties :

1. ionising
2. affect photo film
3. penetrating
4. produces fluorescence
5. can produce photoelectric emission

detection :

1. Geiger-Muller tube
2. scintillation counter
3. solid state detectors
4. photo film

**ultraviolet**generation :

electron energy level changes in atoms

properties :

1. produces ionisation, fluorescence
2. initiates chemical reactions
3. absorbed by plate glass
4. produces the photoelectric effect
5. affects photo film

detection

1. photo film
2. photoelectric cell
3. fluorescent materials

**visible**generation :

electron energy level changes in atoms

properties :

1. starts chemical reactions (eg photosynthesis)
2. affects photo film

detection :

1. photoelectric cell
2. photo film
3. solid state detectors (eg CCD)
4. vision
5. light dependent resistor (LDR)

**infrared**generation :

1. molecular vibration
2. electron energy level changes in atoms

properties :

1. transfer of heat energy to materials
2. modulation for short distance control (eg TV remotes)

detection :

1. CCD devices
2. thermopile
3. special photo film

**microwaves**generation :

1. from magnetrons, klystrons & masers
2. from red-shifted light from stars & galaxies

properties :

1. modulation of waves for communication
2. resonance with molecules, producing heat

detection :

1. directional aerials, parabolic dishes
2. solid state arrays

**radio waves**generation :

1. electrons oscillating
2. red-shifted lower wavelengths from stars etc.

properties :

waves can be modulated for communication

detection :

1. aerials, parabolic dishes
2. solid state arrays

### Absorption & Emission Spectra

Very simply, emission spectra are obtained by viewing light directly. The light is made entirely of emission spectra. That is spectra that are either continuous bands of colour, lines or bunched lines of colour.

On the other hand, absorption spectra are obtained by viewing light through an intervening, translucent substance. Absorption spectra are simply emission spectra with discrete vertical black lines across them.

The explanation is to do with electron energy levels within atoms. Emission spectra are simply discrete wavelengths emitted by atoms when excited electrons fall to lower energy levels.



The wavelength  $\lambda$  is given in terms of the energy level change  $E_2 - E_1$  by the equation:

$$E_2 - E_1 = hf$$

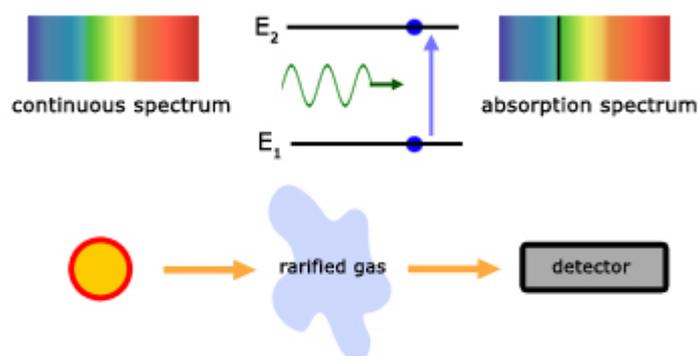
$$v = f\lambda, \quad f = \frac{v}{\lambda}$$

$$\therefore E_2 - E_1 = \frac{hv}{\lambda}$$

where  $h$  is Planck's constant

Note that the energy level change is inversely proportional to the wavelength. In other words, a large energy change produces a short wavelength and vice versa.

When a light beam passes through say a cloud of gas, the electrons in atoms of the gas absorb some of its energy at particular wavelengths. The electrons are then excited to higher energy levels.



Viewing the light after it has passed through the gas reveals black absorption lines. These are evidence of the missing wavelengths of light that were absorbed by the gas atoms.

### Line Spectra

Line spectra are produced by low density, monatomic (single atom) gases and vapours. In this scenario there are no interactions between neighbouring atoms. Both emission and absorption spectra, as described earlier, are examples of line spectra.



a mercury vapour emission spectrum

The black absorption spectra are sometime called **Fraunhofer lines** after **Joseph Von Fraunhofer**, who first discovered them in the solar spectrum.



a section of the solar spectrum

### Band Spectra



spectrum of air

Bands are groups of spectral lines, staggered so they are closer on one side than the other.

Unlike line spectra, bands are produced by molecules, not single atoms. Like line spectra, the material must be in the gaseous or vapour state.

The relative molecular mass(RMM) of the material has some bearing on how close together the lines are.

Low RMM has the effect of spreading lines out.

A high RMM tends to compress them.

### Continuous Spectra

Hot gases emit many wavelengths because they often contain many different kinds of atoms and these are all in different excited states. Atoms also interact between each other as a result of their close proximity. So all the individual emission lines merge to appear as one continuous band of colour.

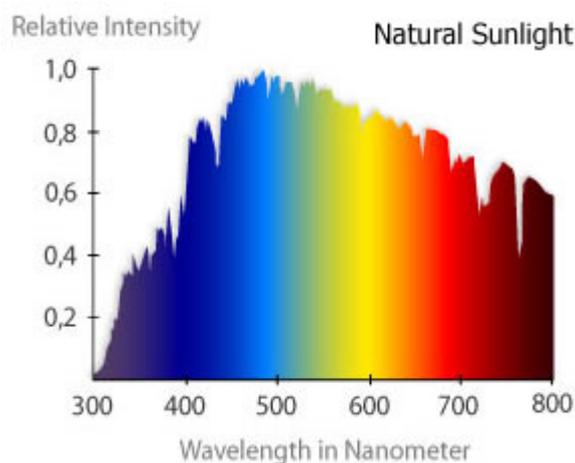


image courtesy of Reef Keeping Fever

Note the jagged nature of the curve. This is a result of absorption caused by the Sun and the Earth's atmosphere.

Interpolating the results, the most intense colour is green. So not surprisingly our eyes are more adapted to that colour.

The overall shape of the curve is that of a 'black body' radiator.

## Stationary(Standing) Waves

### Introduction

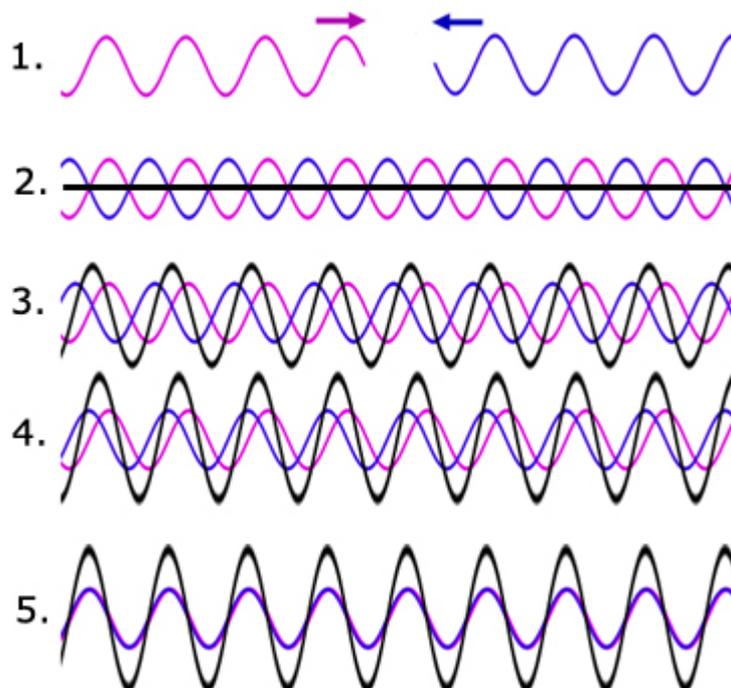
Stationary or Standing waves have become very important in physics in the last hundred years or so. Understanding them has not only given insights into sound but many other important topics eg AC circuit theory, quantum mechanics, nanotechnology.

### Formation of stationary waves

The conditions for standing waves are:

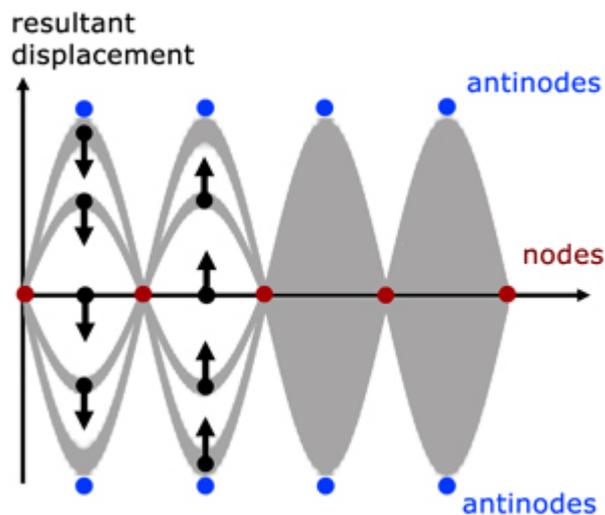
1. two waves travelling in opposite directions along the same line of travel and in the same plane
2. the waves have the same speed
3. the waves have the same frequency
4. the waves have the same approximate amplitude

As a result of **superposition** (waves adding/subtracting), a resultant wave is produced. Now, depending on the phase difference between the waves, this resultant wave appears to move slowly to the right or to the left or disappear completely. It is only when the phase difference is exactly zero and the two waves are exactly in phase, that 'standing/stationary waves' occur.



1. Two waves having the same amplitudes approach each other from opposite directions.
2. The two waves are  $180^\circ$  out of phase with each other and therefore cancel out (black horizontal line).
3. The phase difference between the two waves narrows. The resultant grows but is not in phase with either of the two waves.
4. The phase difference between the two waves is narrower still. The resultant is larger but is still out of phase with the two waves.
5. The phase difference between the two waves is now zero. The resultant has its maximum value and is in phase with the two waves.

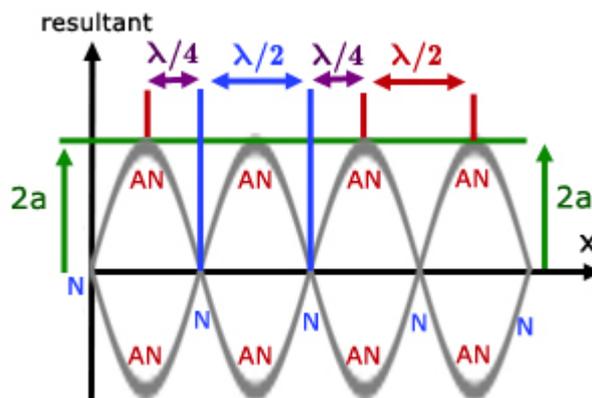
These 'in phase' waves produce an amplitude that is the sum of the individual amplitudes, the region being called an **antinode**. Between two antinodes is a region where the superposition is zero. This is called a **node**.



When the phenomenon is demonstrated with a horizontal vibrating string, the antinode areas appear blurred. To observe the motion of the string moving up and down **astrobe** lamp is used.

### Properties of stationary waves

The diagram shows how a standing wave moves up and down over time.



1. separation of adjacent nodes is half a wavelength ( $\lambda/2$ )
2. separation of adjacent antinodes is also  $\lambda/2$
3. hence separation of adjacent nodes and antinodes is  $\lambda/4$
3. the maximum amplitude is  $2a$  (twice that of a single wave)
4. a standing wave does not transfer energy (its two components however, do transfer energy in their respective directions)

### Stationary wave theory

Consider two waves, R and L, travelling in opposite directions. Their displacements  $y_R$  and  $y_L$  are given by\*:

$$y_R = a \sin(2\pi ft - kx)$$

$$y_L = a \sin(2\pi ft + kx)$$

$$\text{where, } k = \frac{2\pi}{\lambda}$$

\*A derivation of the form of this equation will be provided under '**Derivations - for a deeper understanding**' on the Waves menu at some future date.

When the two waves are superposed, the resultant displacement  $y_T$  is given by:

$$y_T = y_L + y_R$$

$$y_T = a \sin(2\pi ft - kx) + a \sin(2\pi ft + kx)$$

From double angle trigonometry, using one of the 'Factor Formulae' :

$$\sin(C - D) + \sin(C + D) = 2 \sin(C) \cos(D)$$

Comparing this with the expression for  $y_T$ , it is apparent that  $C = 2\pi ft$  and  $D = kx$ .

Therefore,

$$y_T = 2a \sin(2\pi ft) \cos(kx)$$

If we now make,

$$A = 2a \cos(kx)$$

Then  $y_T$  can be rewritten in a form similar to that of a simple sine wave  $y = a \sin(2\pi ft)$

$$y_T = A \sin(2\pi ft)$$

The term  $A$  takes on the significance of being the vertical displacement of the standing wave. From the expression for  $A$  it can be seen that the magnitude of  $A$  depends on the lateral position  $x$ .

Consider the magnitude of  $A$  at different horizontal displacements ( $x$ ) along the standing wave.

$$A = 0 \text{ at a node, } A = \pm 2a \text{ at an antinode}$$

## Beats

### Description

Beats is a phenomenon associated with sound waves, though the effect applies to all waves.

Essentially, when two similar frequencies ( $f_1$ ,  $f_2$ ) are sounded, a third much lower frequency is heard at the same time.

This third frequency is called the **beat frequency** ( $f_B$ ).

The beat frequency is simply the difference between the two original frequencies.

$$f_1 - f_2 = f_B$$

The beat frequency is measured from the rise and fall in the loudness/volume.

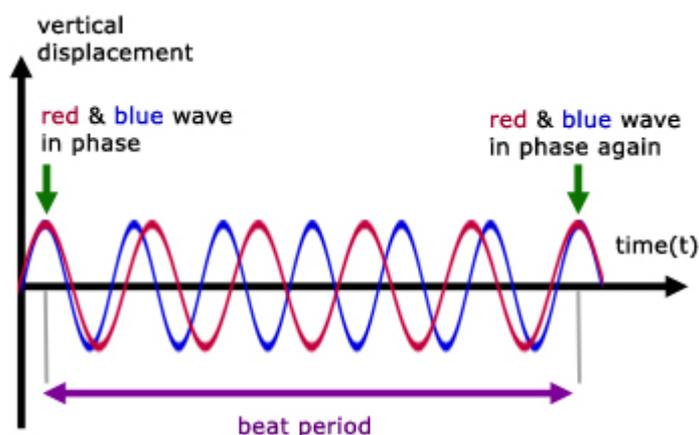
There is yet another frequency, the **combined frequency** ( $f_c$ ). This is the result of superposition of the two original frequencies. The combined frequency is simply the average of these frequencies.

$$f_c = \frac{f_1 + f_2}{2}$$

Since the frequencies  $f_1$ ,  $f_2$  are almost the same, the change in frequency to  $f_c$  is hardly noticeable.

An example of the effect is the sound from a twin engined prop. aircraft. There is a periodic 'wow' or 'drone' noise produced as a result of the change in r.p.m. from the different propeller blades.

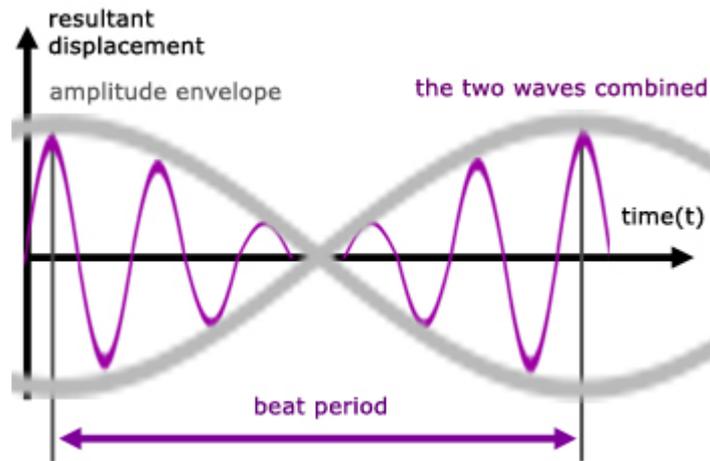
### Explanation



The effect is a result of superposition of two sound wave frequencies producing a succession of constructive and destructive interference.

When the two frequencies are in phase they add, producing a wave with double the amplitude.

When the two waves are out of phase, they destroy each other.



### Theory

Consider our two original frequencies  $f_1$  and  $f_2$ .

In time  $t$  the number of cycles completed by each frequency is  $f_1 t$  and  $f_2 t$  (no. cycles = no. cycles per second  $\times$  no. seconds).

Let us choose the time  $t$  such that the first wave completes one more cycle than the second.

$$f_1 t - f_2 t = 1 \quad t(f_1 - f_2) = 1$$

$$\therefore t = \frac{1}{f_1 - f_2}$$

From the first of two images (above),  $t$  is the time interval between the waves being in phase with each other.

So  $t$  is the **beat period**  $T$  (time for one complete 'beat' wave).

$$T = \frac{1}{f_1 - f_2}$$

For any wave, period and frequency are inversely proportional to one another.

So for beat period  $T$  and beat frequency  $f_B$ ,

$$T = \frac{1}{f_B}$$

hence, by similarity between the last two equations,

$$f_B = f_1 - f_2$$

assuming  $f_1 > f_2$

Measuring an unknown frequency

The method is to use a frequency(  $f_U$  ), where only an approximate value is known. This is used with a known frequency(  $f_K$  ) close to the approximate value of  $f_U$  to produce beats.

The beat frequency (  $f_B$  ) is given by:

$$f_B = f_U - f_K$$

or (depending on the relative magnitudes of  $f_K$  and  $f_U$  )

$$f_B = f_K - f_U$$

bringing the two equations together,

$$f_U = f_K \pm f_B$$

This is quite an accurate method, achieving results of 0.01% accuracy.

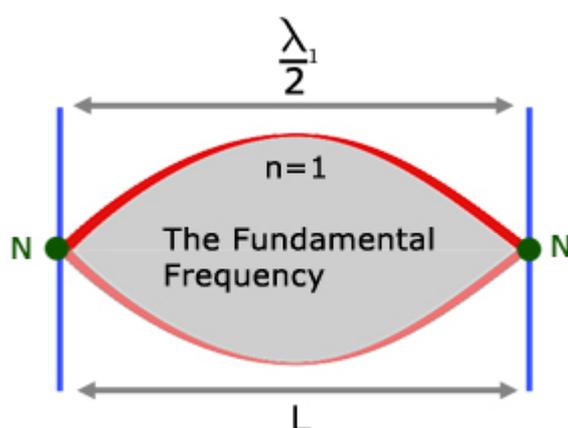
## Waves in Strings

### The Fundamental Frequency

If we consider a length of string with one end tethered, a wave can be sent from the other end by waving the string up and down. The wave reflects at the tethered end and proceeds in the opposite direction.

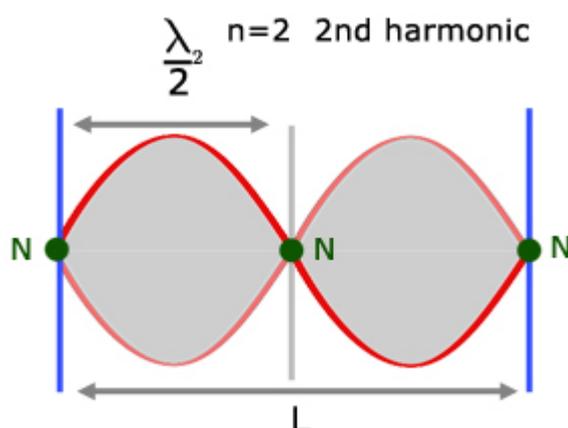
Consider now a continuous wave being produced. The wave travelling to the left interferes with the reflected wave moving to the right.

In this way 'standing waves' are set up. The **Fundamental Frequency** is simply the lowest frequency for a standing wave to form.

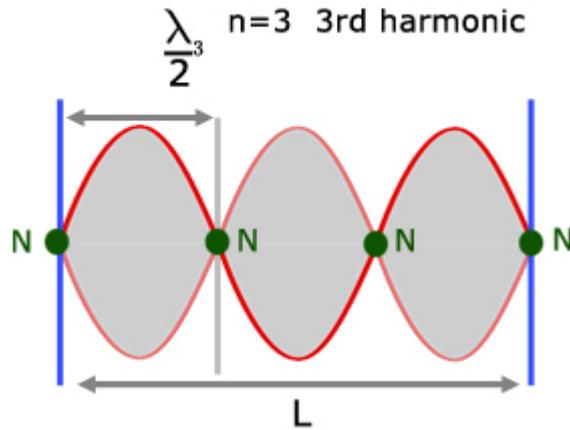


The Fundamental Frequency is just one of a series of particular frequencies called **overtones** or **harmonics**, where standing waves form.

### Harmonics and Overtones



The Fundamental Frequency is called the 1st harmonic. Successive frequencies where standing waves are produced are called the 2nd harmonic, the 3rd harmonic and so on.



Similarly, the higher frequencies above the Fundamental are termed overtones. The next highest frequency above the Fundamental is called the 1st overtone. The next highest after that is the 2nd overtone etc.

So the 2nd harmonic is the 1st overtone.  
The 3rd harmonic is the 2nd overtone etc.

From the diagrams it can be seen that there is a pattern connecting the wavelength ( $\lambda$ ) and the length of the string ( $L$ ).

$$\frac{\lambda_1}{2} = L \quad \frac{\lambda_2}{2} = \frac{L}{2} \quad \frac{\lambda_3}{2} = \frac{L}{3}$$

For the  $n$ th harmonic, the wavelength  $\lambda_n$  is given by:

$$\frac{\lambda_n}{2} = \frac{L}{n}$$

### Frequency theory

First, let us use the familiar wave equation linking velocity, wavelength and frequency,

$$v = f\lambda \quad f = \frac{v}{\lambda}$$

Hence the frequency of the  $n$ th harmonic ( $f_n$ ) is given by:

$$f_n = \frac{v}{\lambda_n} \quad (i)$$

where  $\lambda_n$  is the wavelength of the  $n$ th harmonic and  $v$  is the velocity of the wave in either direction.

From the previous section, the wavelength  $\lambda_n$  is given by:

$$\frac{\lambda_n}{2} = \frac{L}{n}$$

hence,

$$\lambda_n = \frac{2L}{n} \qquad \frac{1}{\lambda_n} = \frac{n}{2L}$$

substituting for  $1/\lambda_n$  into equation (i

$$f_n = \frac{nv}{2L} \qquad \text{(ii)}$$

With  $n=1$ , frequency of the 1st harmonic (the Fundamental)  $f_1$  is given by:

$$f_1 = \frac{v}{2L}$$

Substituting for  $v/2L$  into equation (ii), we obtain the frequency of the nth harmonic in terms of the Fundamental frequency.

$$f_n = nf_1$$

Thus proving that subsequent harmonics are all multiples of the Fundamental Frequency.

#### Effect of mass/unit length, length, tension on frequency

By experiment, it can be shown that,

$$v = \sqrt{\frac{T}{\mu}} \qquad \text{(iii)}$$

where,

$T$  is the tension in the string - Newtons (N)

$\mu$  is the mass/unit length of the string - ( $\text{kgm}^{-1}$ )

From equation (ii) above,

$$f_n = \frac{nv}{2L}$$

making  $v$  the subject,

$$v = \frac{2Lf_n}{n}$$

substituting for  $v$  from equation (iii) above,

$$\frac{2Lf_n}{n} = \sqrt{\frac{T}{\mu}}$$

making  $f_n$  the subject,

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

From the equation it can be seen that:

$$\begin{aligned} f_1 &\propto \frac{1}{L} && T, \mu \text{ constant} \\ f_1 &\propto \sqrt{T} && L, \mu \text{ constant} \\ f_1 &\propto \frac{1}{\sqrt{\mu}} && T, L \text{ constant} \end{aligned}$$

The proportionalities are often termed the **Laws of Vibration for Stretched Strings**.

In simple terms,

long strings make low frequencies and vice versa;

tight strings make high frequencies and vice versa;

thick, heavy strings make low frequencies and vice versa .

## Waves in Pipes

Pipes produce standing waves similar to stretched strings. However it must be emphasized that in stretched strings the waves are transverse, while in pipes the waves are longitudinal.

**Transverse** - particles of the wave vibrate at right angles to the direction of travel of the wave.

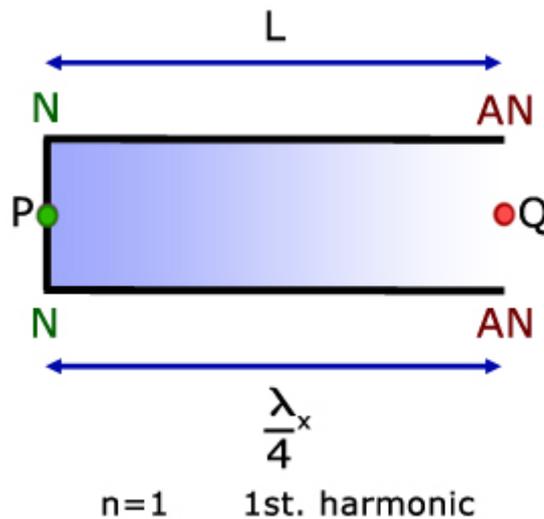
**Longitudinal** - particles of the wave vibrate in the same line as the direction of travel.

### Closed pipes

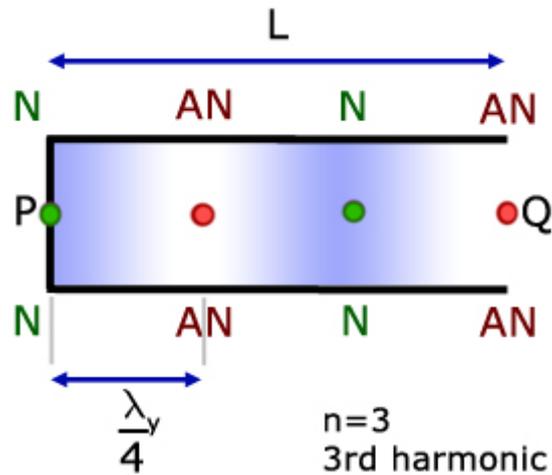
In the diagrams, P is the site of a node, while Q is at an antinode.

Nodes are always formed at the closed end of a pipe, where the air cannot move. Antinodes are always formed at the open end of pipes.

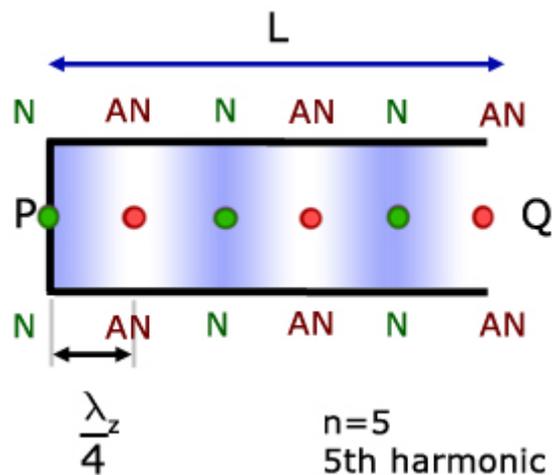
As with stretched strings, the distance between node and antinode is  $1/4$  of a wavelength.



The diagram above represents the Fundamental Frequency, where  $n=1$ . This is the 1st harmonic.



The diagram above represents the 3rd harmonic, sometimes called the First Overtone.



The diagram above represents the 5th harmonic, sometimes called the Second Overtone.

Looking at the different wavelengths in terms of the length of the pipe  $L$ ,

$$\frac{\lambda_x}{4} = L \quad \frac{\lambda_y}{4} = \frac{L}{3} \quad \frac{\lambda_z}{4} = \frac{L}{5}$$

we can then make wavelength the subject of each equation.

$$\lambda_x = 4L \quad \lambda_y = \frac{4L}{3} \quad \lambda_z = \frac{4L}{5}$$

Using the wave equation and making the frequency  $f$  the subject:

$$v = f\lambda$$

$$f = \frac{v}{\lambda}$$

Substituting the different values of wavelength to obtain different expressions for frequency:

$$f_x = \frac{v}{4L} \quad f_y = \frac{3v}{4L} \quad f_z = \frac{5v}{4L}$$

Looking at the form of these equations it is observed that each is a multiple of  $f_x$  (the Fundamental Frequency).

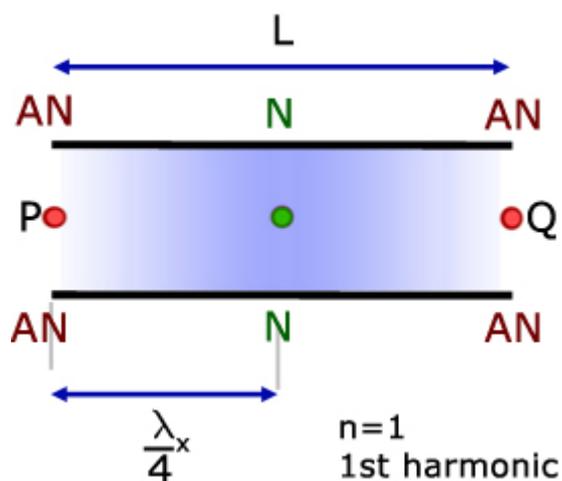
$$f_y = 3 \left( \frac{v}{4L} \right) = 3f_x \quad f_z = 5 \left( \frac{v}{4L} \right) = 5f_x$$

$$f_n = n \left( \frac{v}{4L} \right)$$

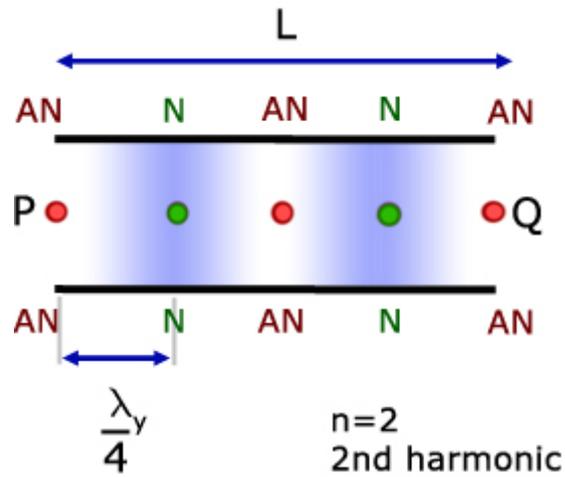
$$\underline{f_n = nf_x}$$

where  $n$  is 1, 3, 5, ... (odd)

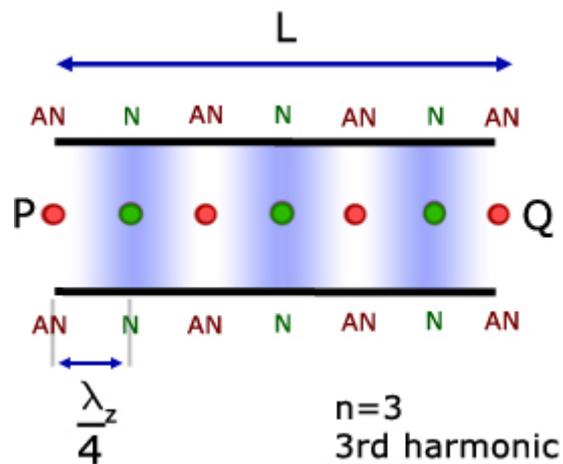
### Open pipes



The diagram above represents the Fundamental Frequency, where  $n=1$ . This is the 1st harmonic.



The diagram above represents the 2nd harmonic, sometimes called the First Overtone.



The diagram above represents the 3rd harmonic, sometimes called the Second Overtone.

Looking at the different wavelengths in terms of the length of the pipe  $L$ ,

$$\frac{\lambda_x}{4} = \frac{L}{2} \quad \frac{\lambda_y}{4} = \frac{L}{4} \quad \frac{\lambda_z}{4} = \frac{L}{6}$$

We can then make wavelength the subject of each equation.

$$\lambda_x = 2L \quad \lambda_y = L \quad \lambda_z = \frac{2L}{3}$$

Using the wave equation and making the frequency  $f$  the subject:

$$v = f\lambda \qquad f = \frac{v}{\lambda}$$

We can now substitute the different values of wavelength to obtain different expressions for frequency:

$$f_x = \frac{v}{2L} \qquad f_y = \frac{v}{L} \qquad f_z = \frac{3v}{2L}$$

Looking at the form of these equations it is observed that each is a multiple of  $f_x$  (the Fundamental Frequency).

$$f_y = 2 \left( \frac{v}{2L} \right) = 2f_x \qquad f_z = 3 \left( \frac{v}{2L} \right) = 3f_x$$

$$f_n = n \left( \frac{v}{2L} \right)$$

$$\underline{f_n = nf_x}$$

where  $n$  is 1, 2, 3, 4, 5, ... (odd + even)

#### A comparison of 'closed' and 'open' pipes

1. Comparing expressions for the Fundamental Frequency ( $n=1$ ) for closed and open pipes respectively,

$$f_{1_{closed}} = \frac{v}{4L} \qquad f_{1_{open}} = \frac{v}{2L}$$

For a pipe of the same length  $L$ , the open pipe frequency is twice that of the closed pipe frequency.

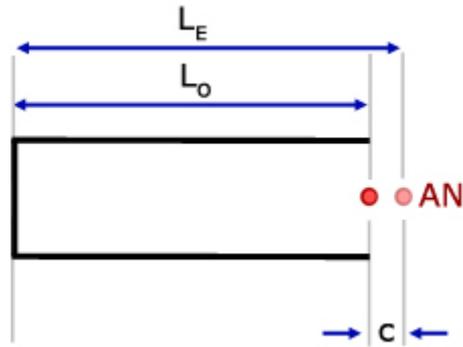
$$f_{1_{open}} = 2f_{1_{closed}}$$

2. For a given length of pipe, an open pipe gives more harmonics (odd & even) than a closed pipe (odd only). This results in a richer note from the open pipe.

### End correction

The 'end correction' ( $c$ ) is a length that must be added on to the the length ( $L_o$ ) of a pipe to take account of antinodes extending beyond the open end of the pipe.

End correction for a closed pipe



The effective length ( $L_E$ ) is given by:

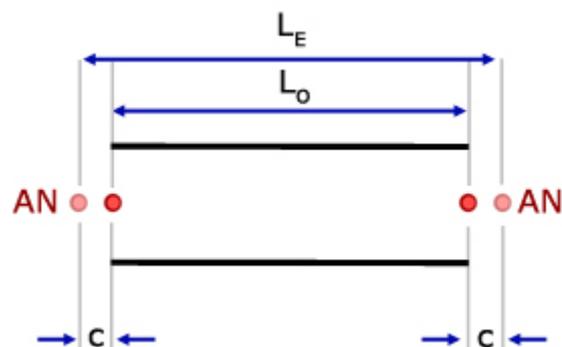
$$L_E = L_o + c$$

$$c = 0.6r$$

$$\underline{L_E = L_o + 0.6r}$$

where  $r$  is the radius of the pipe

End correction for an open pipe



The effective length ( $L_E$ ) is given by:

$$L_E = L_O + 2c$$

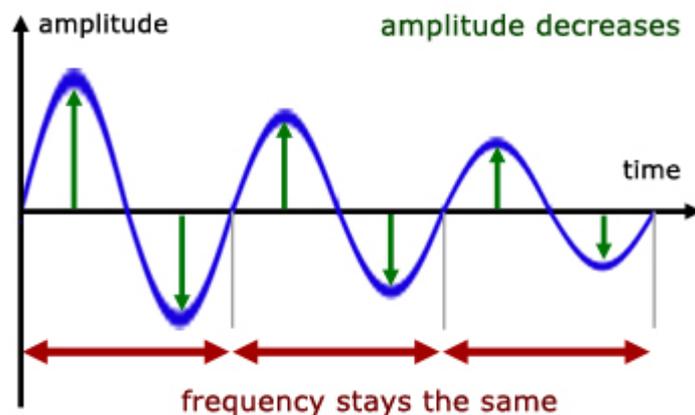
$$c = 0.6r$$

$$\underline{L_E = L_O + 1.2r}$$

where  $r$  is the radius of the pipe

## Resonance

### Damping

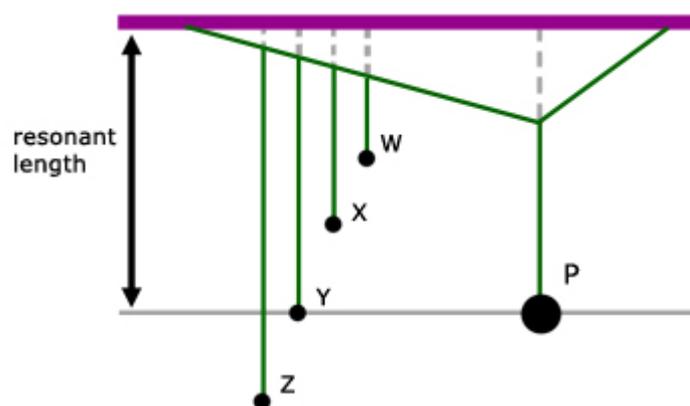


Damping describes the loss of energy of an oscillatory motion. The loss of energy is evident in the reduction in amplitude of the wave. Successive waves become smaller, however, the frequency remains the same.

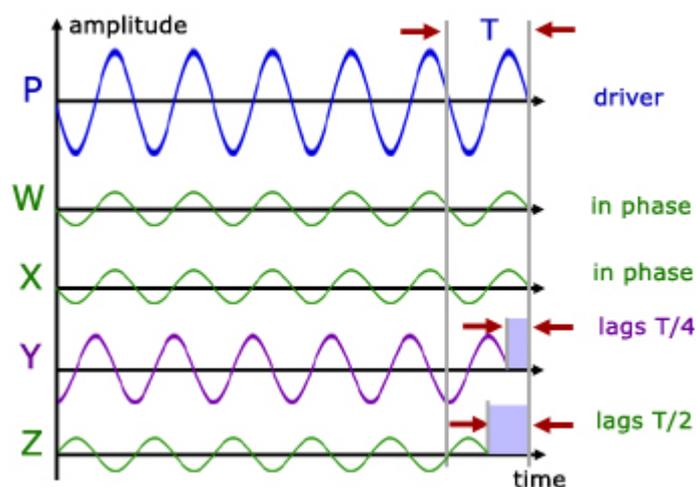
One common example is the pendulum. After being set in motion, the distance being swept out by the pendulum bob becomes progressively smaller. Energy is lost to the system due to air resistance and friction at the support.

### Forced vibrations

**Forced vibration** can be illustrated by a simple experimental setup called Barton's pendulums.



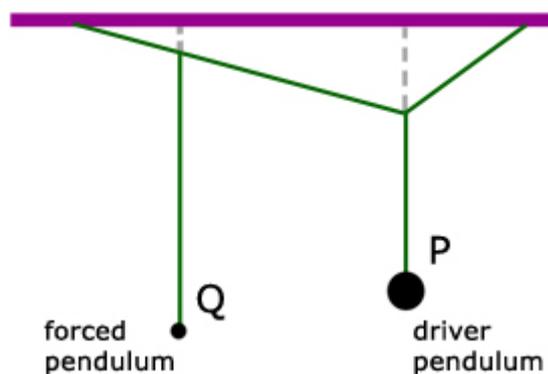
Essentially a large pendulum is used to provide a **driver frequency** that will make the other, smaller pendulums oscillate at the same rate. This driver frequency is in fact the **natural frequency** of the pendulum. That is, the frequency at which it would oscillate without the smaller pendulums. The masses of the smaller pendulums are insignificant compared to the mass of the larger. So their effect is not of any consequence.



On the graph it should be noted:

1. All the pendulums have the same frequency.
2. Pendulum Y has the same length as P but is not in phase with it. It is approx.  $1/4$  period behind. However, Y does oscillate with greater amplitude than the other small pendulums. Y is said to be **resonating** with P.
3. The shorter pendulums, W & X are approx. in phase with P.
4. Pendulum Z is approx.  $1/2$  period behind P.

Using Barton's pendulums to investigate forced vibrations



The method is to keep the length of the dependent pendulum Q the same, while varying the length of the driver pendulum P.

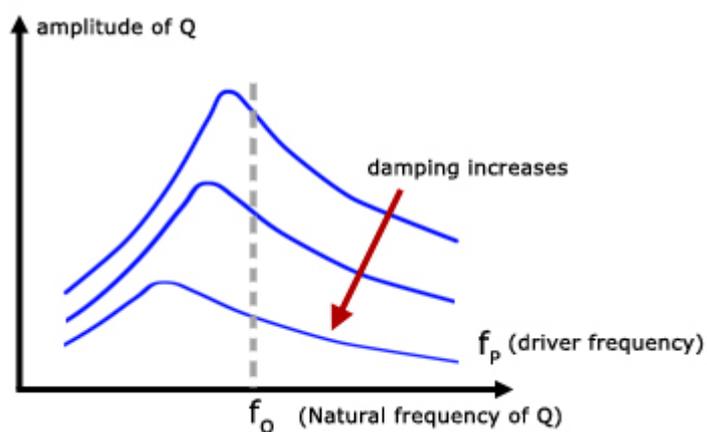
Measurements are then made of:

- 1) the frequency of P
- 2) the frequency of Q
- 3) the amplitude of Q
- 4) the phase difference between P and Q

Initial graphs (blue) are drawn for the amplitude of Q and the phase difference of Q against driver frequency.

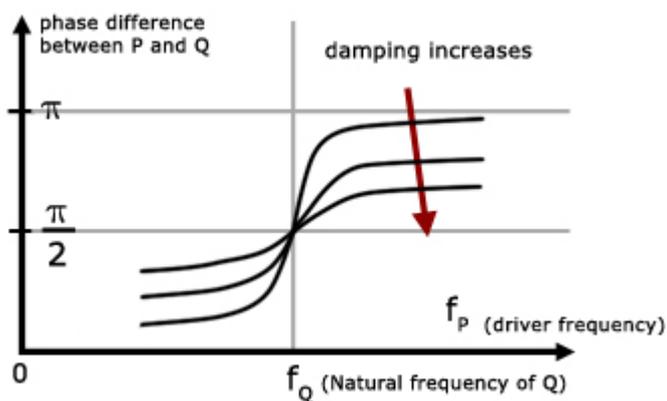
Various levels of damping are then applied to Q. Families of curves are produced.

More damping is produced by decreasing the bob mass of Q and/or adding cardboard fins to increase air resistance.



It can be seen that the amplitude of pendulum Q is maximum (maximum resonance) just before the natural frequency of the driver pendulum P.

If Q is heavily damped the amplitude is much less at its maximum and occurs at a much lower frequency than the natural driver frequency.



Whether pendulum Q has light or heavy damping, it always has a phase difference of  $\pi/2$  radians ( $90^\circ$ ) with P, at the natural frequency with which P oscillates.

For higher driver frequencies and light damping the phase difference rises to a maximum of  $\pi$  (pi) radians ( $180^\circ$ ).

Higher driver frequencies and heavy damping produce a phase difference only slightly above  $\pi/2$  radians.

### implications & uses of resonance

implications:

1. Soldiers must 'break step' when crossing wooden bridges.
2. Cars/aircraft/rockets are carefully designed so that parts do not resonate producing unwanted noises/dangerous vibrations.
3. Electrical audio circuits are subject to 'feedback' . This is the loud howling sound produced when a microphone is too close to a loudspeaker and the amplifier gain is too high.

uses:

1. clocks & watches - quartz crystals resonate producing accurate timing frequencies
2. standing waves in pipes
3. ultrasonic cleaning - dirt particles resonate with the applied frequency and are dislodged
4. crystal radios - circuits resonate at the same frequency as a radio station
5. radio antennas (aerials) - resonate when they interact with radio waves

## The Doppler Effect

### Description

The effect is caused by the relative motion of an observer and a source of waves.

The observed frequency (from the observer's viewpoint) is different from the actual frequency. The actual frequency is the frequency emitted from the source.

### Derivation of frequency change

To aid understanding, the derivation is best broken down into a number of sections:

**observed wavelength - static observer in front of moving source**

**observed wavelength - static observer behind moving source**

**observed frequency - moving observer forward of source, moving towards it**

**observed frequency - moving observer behind source, moving towards it**

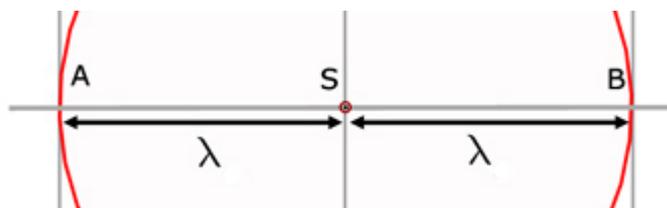
Consider a stationary source of waves S.

A wave is emitted and one period later another wave is just about to be produced.

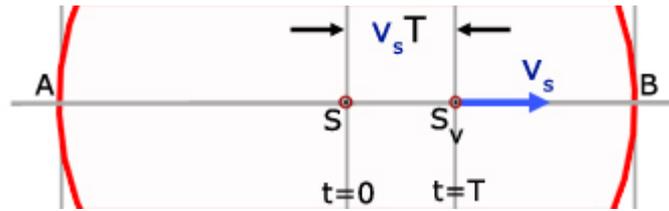
The separation of successive crests is the wavelength  $\lambda$ .

From the wave equation,

$$c = f\lambda \quad \lambda = \frac{c}{f}$$



observed wavelength - static observer in front of moving source



The source moves towards B at velocity  $v_s$ .

In the time span of one period ( $T$  secs.) the source moves towards **B** a distance  $v_s T$  (distance = velocity  $\times$  time) to  $S_v$ .

in this time, the first wave has moved from **S** to **B**.

The source is considered to carry the second wave, which is on the point of being emitted.

The distance of the crest of the 1st wave at **B**, measured from **S** is  $d_{w1}$ .

The distance of the crest of the 2nd wave at  $S_v$ , measured from **S** is  $d_{w2}$ .

$$d_{w1} = cT \qquad d_{w2} = v_s T$$

So the distance between crests is given by:

$$d_{w1} - d_{w2} = cT - v_s T \qquad (i)$$

Remembering that,

$$T = \frac{1}{f}$$

we can substitute for  $T$  into (i)

$$\begin{aligned} d_{w1} - d_{w2} &= c \left( \frac{1}{f} \right) - v_s \left( \frac{1}{f} \right) \\ &= \frac{c}{f} - \frac{v_s}{f} \\ &= \frac{c - v_s}{f} \end{aligned}$$

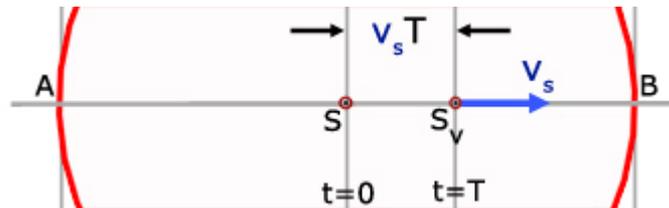
If  $\lambda_F$  is the wavelength forward of the crests,  
(the distance between the crests)

then,

$$\lambda_F = d_{W1} - d_{W2}$$

$$\therefore \lambda_F = \frac{c - v_s}{f}$$

observed wavelength - static observer behind moving source



This is the same diagram as for the observer in front of the source. But look again, this time from the left at **A**, looking towards the moving source.

This time we consider the distance between the wave at **A** and the wave about to be emitted at **S<sub>v</sub>**.

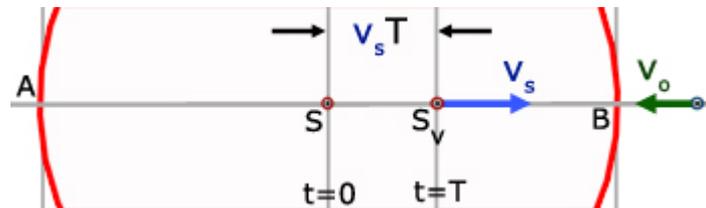
Here the distance moved by the first wave from **S** and the distance moved by the source are **added**.

$$\begin{aligned} d_{W1} + d_{W2} &= \frac{c}{f} + \frac{v_s}{f} \\ &= \frac{c + v_s}{f} \end{aligned}$$

$$\lambda_B = d_{W1} + d_{W2}$$

$$\lambda_B = \frac{c + v_s}{f}$$

derivation of frequency - moving observer forward of source, moving towards it



$v_o$  is the velocity of an observer moving towards the source.

This velocity is independent of the motion of the source.

Hence, the velocity of waves relative to the observer is  $c + v_o$ .

The wavelength observed forward of the source is therefore given by:

$$\lambda_F = \frac{c + v_o}{f_F}$$

Recalling the equation for 'a static observer in front of a moving source':

$$\lambda_F = \frac{c - v_s}{f}$$

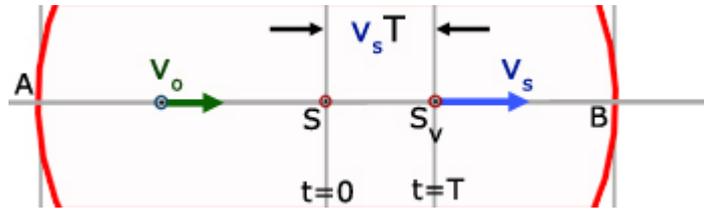
equating these two,

$$\frac{c - v_s}{f} = \frac{c + v_o}{f_F}$$

to make  $f_F$  (frequency forward of the source) the subject

$$f_F = \left( \frac{c + v_o}{c - v_s} \right) f$$

derivation of frequency - moving observer behind source, moving towards it



Remembering that  $v_o$  is independent of the motion of the source, the wavelength observed forward of the source is given by:

$$\lambda_B = \frac{c + v_o}{f_B}$$

Recalling the equation for 'a static observer behind a moving source':

$$\lambda_B = \frac{c + v_s}{f}$$

equating these two,

$$\frac{c + v_s}{f} = \frac{c + v_o}{f_B}$$

to make  $f_B$  (frequency behind the source) the subject

$$f_B = \left( \frac{c + v_o}{c + v_s} \right) f$$

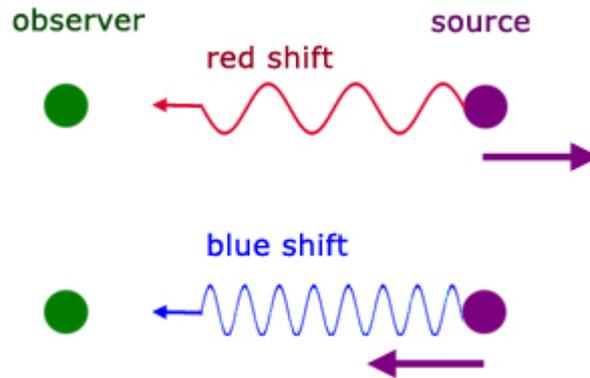
important conclusions:

1. For an observer moving away from the source, the value of  $v_s$  is negative.
2. The motion of an observer does not alter the wavelength. The increase in frequency is a result of the observer encountering more wavelengths in a given time.
3. When the source is stationary ( $v_s = 0$ ),  $f_A = f_B$ . So it makes no difference whether the observer is in front or behind the source.

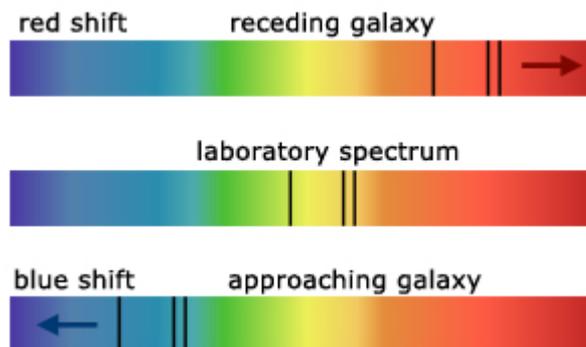
### Application in astronomy

A star or galaxy having a **red shift** means that the celestial object is moving away from us. Light rays become more spread out.

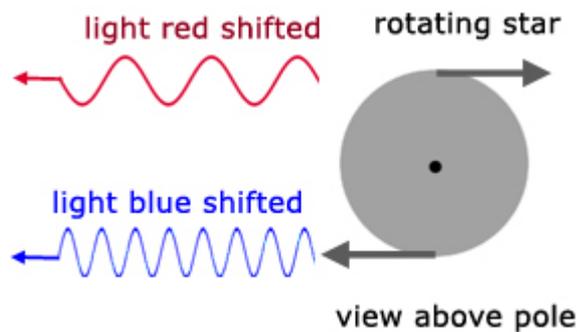
A **blue shift** indicates the opposite. The object is moving towards us. Light rays become bunched up.



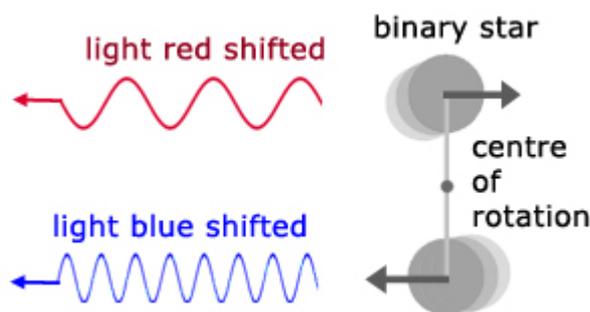
The effects are descriptions of how emission and absorption patterns in spectra are moved either towards the blue or the red end of the spectrum. The movement is apparent when spectra are compared with spectra produced in the lab.



The rotation of celestial objects (eg the Sun, Saturn's rings) can be measured from the light moving towards and away from us at the limb.



Similarly, double stars are detected by their star-light containing both red shifted and blue shifted lines. This is a result of one star approaching the observer, while the other is receding (a consequence of both stars revolving around a common centre).



### Radar speed traps

The speed of an approaching car, or one speeding away, can be found from the change in frequency of microwaves reflected from it.

The following relation is used to calculate  $v_{car}$  the velocity of an approaching/receding car.

$$v_{car} = \frac{cf_{beat}}{2f}$$

where,

**c** is the speed of light

**f** is the frequency of the microwaves

**f<sub>beat</sub>** is the *beat frequency* produced by interference between the original and reflected waves

### Plasma temperature

When the spectrum of a hot plasma is examined, the spectral lines are observed to broaden with increased temperature.

This is because atoms emitting light are moving away from an observer and at the same time coming towards him/her.

Plasmas are extremely hot gases with temperatures in excess of  $10^6$  deg. Celsius.

The light emitted from an excited atom dropping to a particular energy state would normally be of **one** discrete wavelength. However, the action of the Doppler effect means that wavelengths of slightly longer and shorter wavelengths are emitted. Hence the spectral line is broadened by the extra wavelengths being produced.

**Spectral line broadening is proportional to the square root of the absolute temperature.** So by measuring the broadening the temperature can be found.