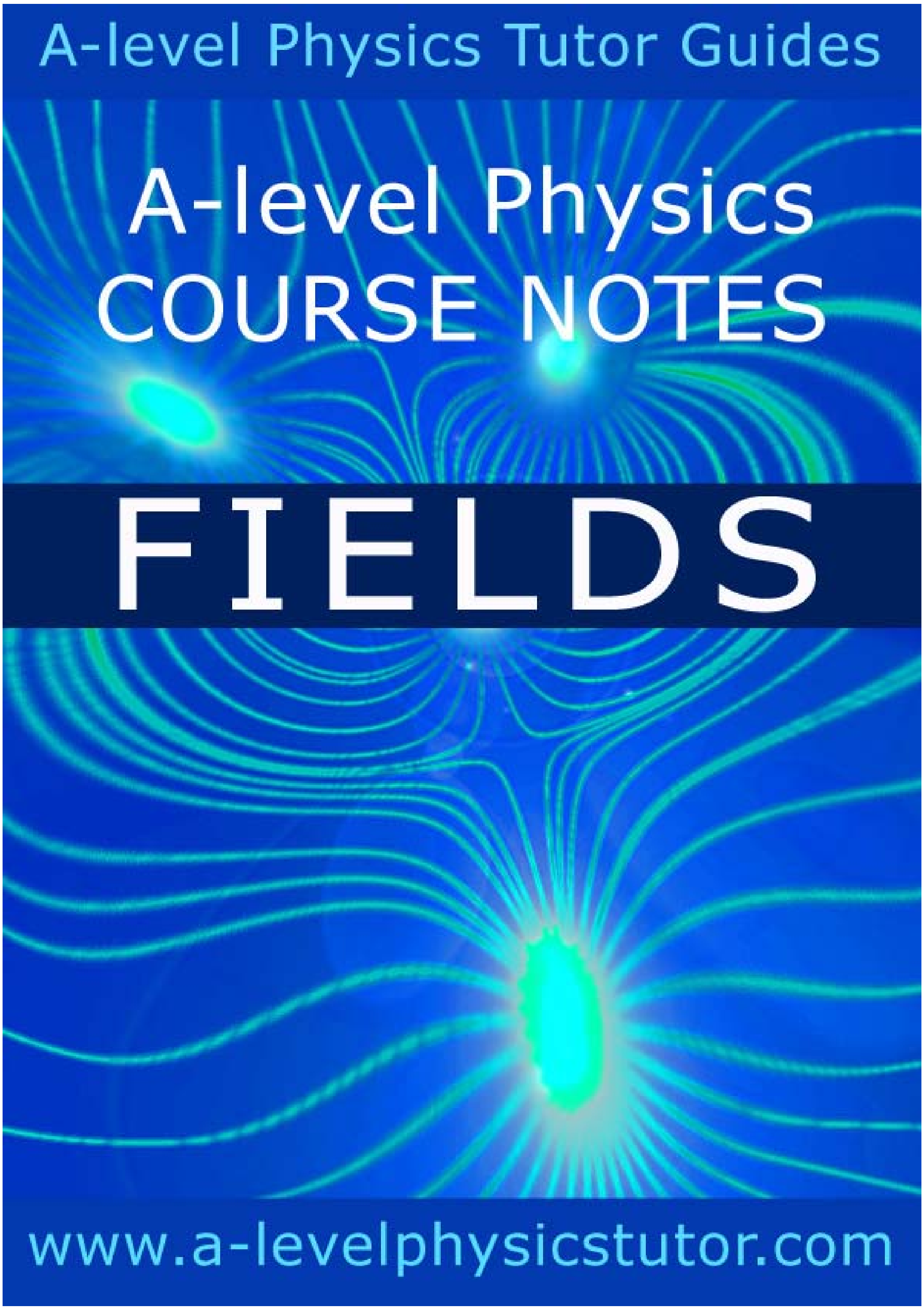
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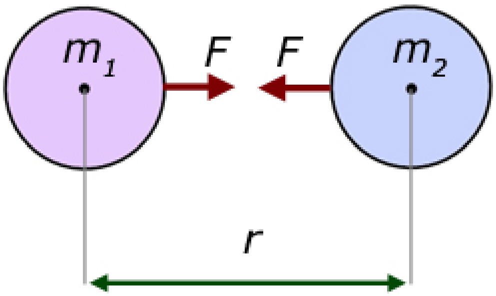
Gravitational Fields - Part 1

Newton's Law of Gravitation

**For two masses displaced a distance apart,
the gravitational force of attraction of one mass on the other is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them.**



If the masses are $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$, with their centres of mass displaced a distance ***r*** apart, then the force of attraction ***F*** of one mass on the other is described as:

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{m_1 m_2}{r^2}$$

The proportionality can be made into an equation using a constant of proportionality. This constant we call **G**, the Universal Gravitational Constant.

$$F = G\frac{m_1 m_2}{r^2}$$

$\boldsymbol{G}$ = 6.67 x 10^{-11} N m^2 kg^{-2}

Gravitational force is very weak! This can be shown by considering two 1 kg masses 1 m apart. The gravitational force between them is given by:

$$F = 6.67 \times 10^{-11}\left(\frac{1 \times 1}{1^2}\right)$$
$$= 6.67 \times 10^{-11} N$$

The gravitational force between everyday objects is so small as to be almost irrelevant.

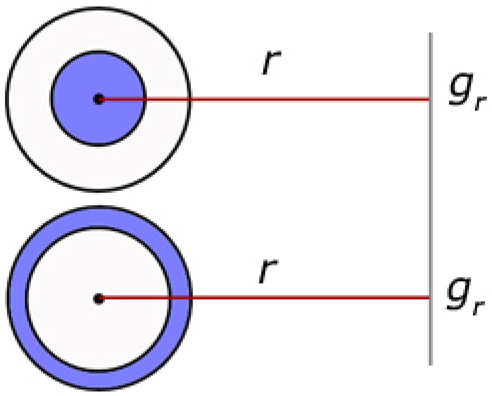
Variation of '*g*' with distance from the Earth's centre

To understand this work we must recall that '***g***' is the acceleration due to gravity (9.8 ms $^{-2}$). This value is for the *surface* of the Earth. Weight ***W*** is the force of attraction of the earth on a mass. For a mass ***m***, the weight is given by:

$$W = mg$$

The mathematical treatment depends on two assumptions:

1. The value of *g* is the same at a distance from a mass, whether the mass is in the shape of a spherical shell or concentrated in the centre.



2. The value of *g* everywhere inside a spherical shell is zero.

NB the spherical shell and central mass have uniform density

First consider a mass ***m*** on the surface of the Earth. The force of attraction between the mass and the earth is its weight ***W***. This is also equal to the force ***F*** between the mass and the Earth, given by Newton's Law.

$$W = F$$

$$mg = G\frac{M_E m}{r_E^2}$$

$$g = G\frac{M_E}{r_E^2}$$ (i

where $\boldsymbol{M_E}$ is the mass of the Earth, $\boldsymbol{r_E}$ its radius

Now let us consider the value of ***g*** at a distance ***r*** from the Earth.

case where *r* > r_E

If this new value is g_r , then by similarity with equation (i,

$$g_r = G\frac{M_E}{r^2}$$ (ii

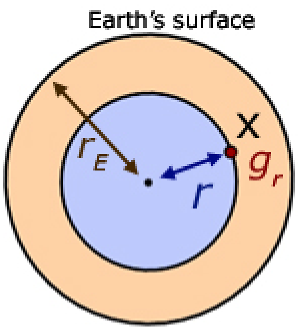
Dividing equation (ii by equation (i,

$$\frac{g_r}{g} = \left(G\frac{M_E}{r^2}\right)\left(\frac{r_E^{\ 2}}{GM_E}\right)$$

$$\frac{g_r}{g} = \left(\frac{r_E^{\ 2}}{r^2}\right)$$

$$g_r = \left(\frac{r_E^{\ 2}}{r^2}\right)g$$ (iii

case where *r* < r_E



In the diagram the point **X** is **inside** the earth at a distance ***r*** from the centre.

From our initial assumptions, the value of g_r is a result of the gravity from a sphere of radius ***r*** .

If M_S is the mass of the sphere, then by comparison with equation (i ,

$$g_r = G\frac{M_S}{r^2}$$

$$M_S = \frac{g_r r^2}{G}$$ (iv

NB the effect of matter (in the form of a shell) above point X has no effect on the value of g_r

let us assume that masses have uniform density ***ρ*** (rho).

Remembering that $\boldsymbol{m = \rho V}$, the mass $\boldsymbol{M_S}$ of the internal sphere and the mass $\boldsymbol{M_E}$ of the Earth is given by:

$$M_S = \frac{4}{3}\pi r^3 \rho$$

$$M_E = \frac{4}{3}\pi r_E{}^3 \rho$$

dividing the first equation by the second,

$$\frac{M_S}{M_E} = \frac{r^3}{r_E{}^3}$$

$$M_S = M_E\left(\frac{r^3}{r_E{}^3}\right)$$

Substituting for $\boldsymbol{M_S}$ from equation (iv ,

$$\frac{g_r r^2}{G} = M_E\left(\frac{r^3}{r_E{}^3}\right)$$

$$g_r = GM_E\left(\frac{r}{r_E{}^3}\right)$$

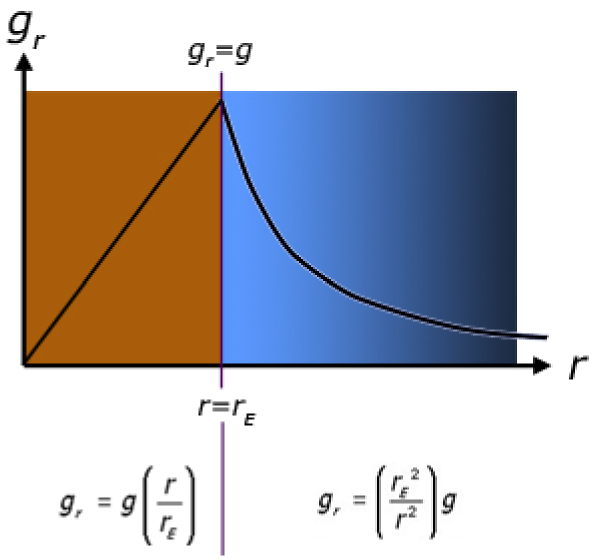
recalling that,

$$g = G\frac{M_E}{r_E{}^2}$$

hence,

$$g_r = \left(G\frac{M_E}{r_E{}^2}\right)\frac{r}{r_E}$$

$$g_r = g\left(\frac{r}{r_E}\right)$$



NB $g_r = g$ when $r = r_E$

Summary

So for **inside the Earth**, g_r is directly proportional to r . The graph is therefore a straight line through the origin.

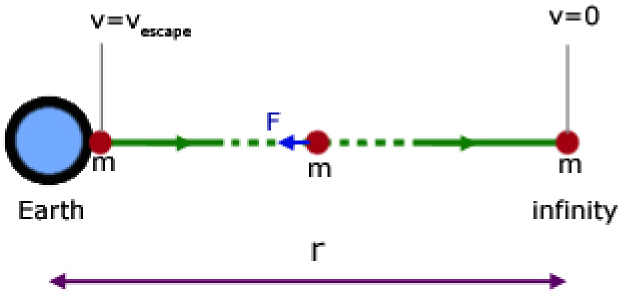
For **outside the Earth**, g_r follows a function similar to $y = x^{-2}$, where x decreases steadily, approaching zero at infinity.

$$g_r = \left(\frac{r_E^{\,2}}{r^2}\right)g$$

where r_E and g are constants

Gravitational Fields - Part 2

Escape velocity



Theoretically (neglecting air resistance) to leave the Earth and not return, a mass must have enough kinetic energy to reach a point an infinite distance away, where its velocity (and hence KE) is zero.

The **escape velocity** is the minimum initial velocity required to do this. This is a constant for a particular planetary mass, and is independent of the projected mass.

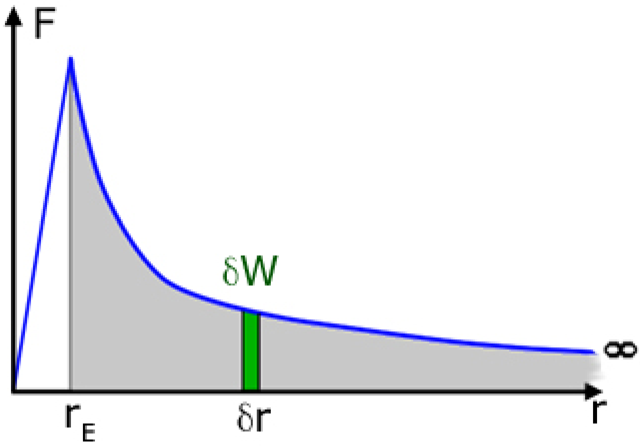
Consider a mass m being projected away from the surface of the Earth with velocity v.

At a distance r from the Earth (mass M_E) the force of gravity F on the mass is given by:

$$F = G\frac{M_E m}{r^2}$$

This equation can be used to calculate the work done by the gravitational force in bringing the mass m to rest.

Consider the mass ***m*** moving an incremental distance ***δr***(delta ***r***) away from the Earth. That is, against the force ***F*** .



Since **work done = force x distance moved against force**, the incremental work ***δW*** done by the gravitational field on the mass is given by;

$$\delta W = F\delta r$$

substituting for ***F*** from the first equation,

$$\delta W = G\frac{M_E m}{r^2}\delta r$$

Making the expression into an integral, where ***W*** is the total work done by the gravitational force between the limits of $r = r_E$ and r = infinity :

(essentially summing the individual slices of ***F δr*** between the limits to obtain the area under the curve)

$$W = \int_{r_E}^{\infty}\left(G\frac{M_E m}{r^2}\right)dr$$

$$W = GM_E m\int_{r_E}^{\infty}\left(\frac{1}{r^2}\right)dr$$

Integrating between the limits,

$$W = GM_E m\left[-\frac{1}{r}\right]_{r_E}^{\infty}$$

$$W = \frac{GM_E m}{r_E}$$

The work done by the gravitational force on the mass equates to the original (maximum) kinetic energy of the mass.

$$\frac{1}{2}mv^2 = \frac{GM_E m}{r_E}$$

Making the velocity ***v*** the subject,

$$v = \left(\frac{2GM_E}{r_E}\right)^{1/2}$$

For the Earth, the escape velocity approximates to 11 kms^{-1}or 7 miles s^{-1}.

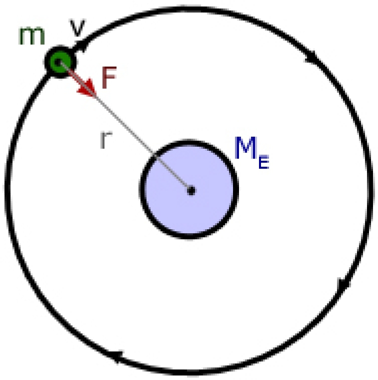
NB for this theoretical treatment
1. the theory does not apply to continuously propelled masses
2. escape velocity is independent of the direction of projection

celestial object	v_{escape} (km/s)
Sun	617.5
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.4
Mars	5.0
Jupiter	59.5
Saturn	35.6
Uranus	21.2
Neptune	23.6
Pluto	3.9
solar system	≥ 525
event horizon	speed of light 3×10^5

Satellite orbits

A satellite (mass ***m)*** orbits the Earth (mass M_E) at a constant velocity ***v*** .

The centripetal force keeping the satellite in orbit is provided by the gravitational force of attraction ***F*** between the mass and the Earth.



The equations for centripetal and gravitational force are combined.
Satellite velocity ***v*** is then made the subject of the equation.

$$F = \frac{mv^2}{r} \qquad F = \frac{GM_E m}{r^2}$$

$$\frac{mv^2}{r} = \frac{GM_E m}{r^2}$$

$$v = \left(\frac{GM_E}{r}\right)^{1/2}$$

Since ***G*** and M_E are constants, satellite velocity is solely dependent on orbital radius.

The period ***T*** of the motion is simply the circumference of the circular orbit divided by the satellite's velocity.

$$T = \frac{2\pi r}{v}$$

$$T = 2\pi r\left(\frac{r}{GM_E}\right)^{1/2}$$

$$T = 2\pi\left(\frac{r^3}{GM_E}\right)^{1/2}$$

Since ***G*** and M_E are constants, orbital period, like orbital velocity, is soley dependent on orbital radius.

Low orbits

For satellites in orbit a distance equal or less than 200 km above the Earth's surface, the radius of the orbit approximates to the radius of the Earth:

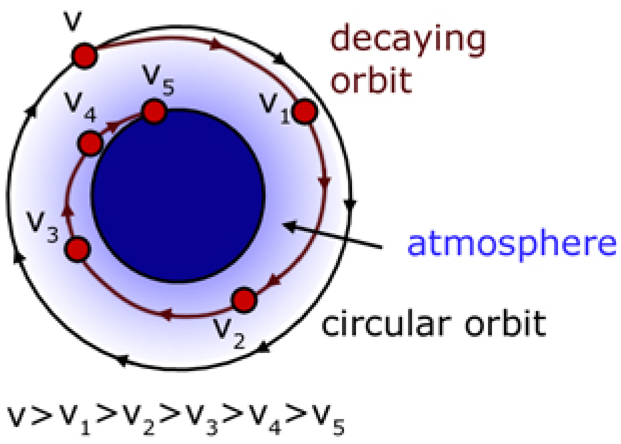
$\boldsymbol{r_E} = 6.6 \times 10^6$ m $\qquad \boldsymbol{r} = 6.8 \times 10^6$ m

Making $\boldsymbol{r}$ equal to $\boldsymbol{r_E}$, the equations for orbital velocity $\boldsymbol{v}$ and period $\boldsymbol{T}$ become:

$$v = \left(\frac{GM_E}{r_E}\right)^{1/2} \qquad T = 2\pi\left(\frac{r_E^{\ 3}}{GM_E}\right)$$

Low Earth orbits are not stable. The outer reaches of the Earth's atmosphere produce drag on a satellite. This changes the satellite's kinetic energy to heat energy, as it is brought back to Earth.

Permanent satellites like the ISS and Hubble have to be given a regular boost to maintain there orbits. Lost kinetic energy is replenished.



note: The Law of Gravitation predicts that lower orbits have higher velocities. So a satellite should go faster and faster as it moves closer to the Earth. However in this case kinetic energy is lost(as heat energy). The atmosphere brakes the motion. So the law does not apply.

Geostationary orbits

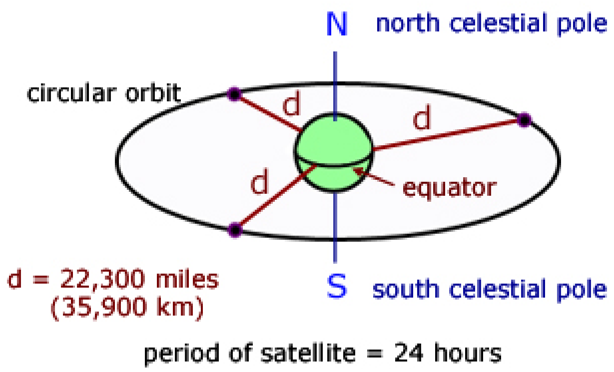
A geostationary satellite is one that always appears in the same place in the sky, no matter what the time of day.

The conditions for this to occur are:

1. the satellite must have an orbital period of exactly 24 hours

2. the satellite must have a circular orbit above the equator

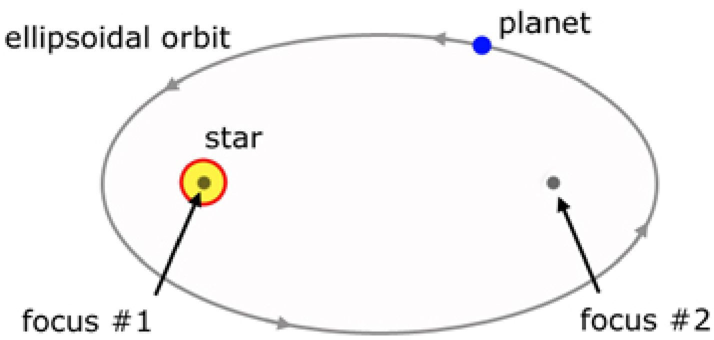
3. the satellite must be orbiting in the same direction as the Earth is rotating



Gravitational Fields - Part 3

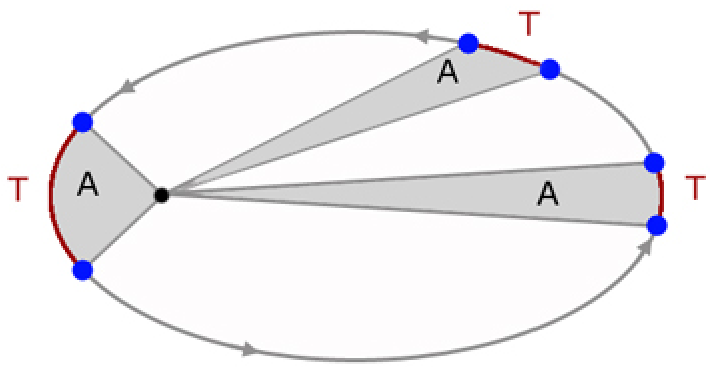
Law #1 - The Orbit Law

The orbit of a planet is in the shape of an ellipse, with the parent star at one focus.

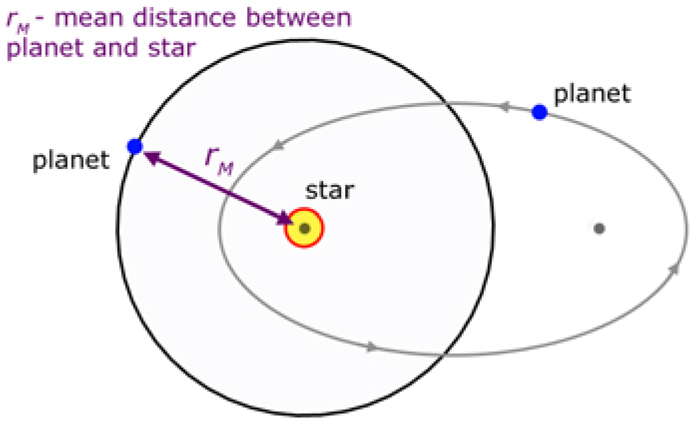


Law #2 - The Area Law

A planet moves such that an imaginary line between it and the parent star sweeps out equal areas in equal times.



Law #3 - The Period law



The square of the orbital period of a planet is proportional to the cube of the mean distance to its parent star.

T - orbital period
$\boldsymbol{r_M}$ - mean distance

$$T^2 \propto r_M{}^3$$

Kepler's 3rd law derived from Newton's Law of Gravitation

The centripetal force ***F*** keeping a mass ***m*** in orbit is given by:

$$F = \frac{mv^2}{r}$$

The angular velocity ***ω*** is the angle (in radians) traced out when the mass travels ***v*** metres in one second. Stating this definition in an equation and making ***v*** the subject:

$$\omega = \frac{v}{r} \qquad v = \omega r$$

Substitung for ***v*** into our equation for centripetal force:

$$F = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r}$$

$$F = m\omega^2 r$$ (i

The centripetal force is provided by gravity.

Hence,

$$F = G\frac{M_s m}{r^2}$$ (ii

Equating equations (i and (ii ,

$$G\frac{M_s m}{r^2} = m\omega^2 r$$ (iii

The period ***T*** of the orbital motion is the circumference (in radians) divided by the angular velocity. Making ***ω*** the subject of the equation:

$$T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T}$$

and substituting for ***ω*** into equation (iii

$$G\frac{M_s m}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r$$

We obtain the expression:

$$G\frac{M_s m}{r^2} = \frac{4m\pi^2 r}{T^2}$$

Now making T^2 the subject:

$$T^2 = \frac{4\pi^2 r^3}{GM_s}$$

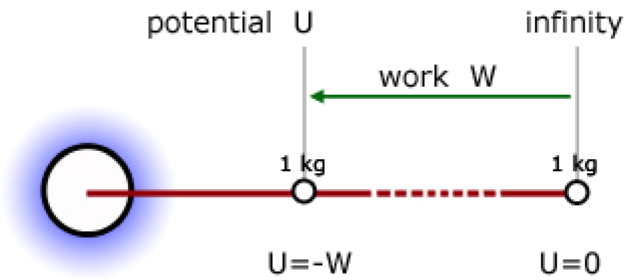
If we now remove the constants ***G M_S π*** by making the equation a proportionality:

$$T^2 \propto r^3$$

Hence Kepler's 3rd Law is consistent with Newton's Law of Gravitation.

Gravitational Fields - Part 4

Gravitational potential - *U*



The potential ***U*** at a point in a gravitational field is defined as being numerically equal to the work done by the field in bringing a unit mass from infinity to the point.

By definition, the potential at infinity is zero.

$$U = \frac{W}{m}$$

where,

U is the gravitational potential at a point

W is the work done in bringing a mass ***m*** from infinity to that point.

The units for gravitational potential (work/mass) are $\mathbf{Jkg^{-1}}$.

In the treatment of **escape velocity**(Gravitational Fields - Part 2) the work done in moving a mass ***m*** from the surface of the Earth to infinity was given by:

$$W = \frac{GM_E m}{r_E}$$

Now if we make the mass unity(m = 1 kg), the energy difference between the 1 kg mass on the surface of the Earth and at infinity (zero potential) is ***W .***

However, since the highest potential is zero at infinity, all potential energies relative to this level are less than zero (ie negative).

Our 1 kg mass on the Earth therefore has a potential of **- *W*** .

If U_E is the potential on the surface of the earth, then:

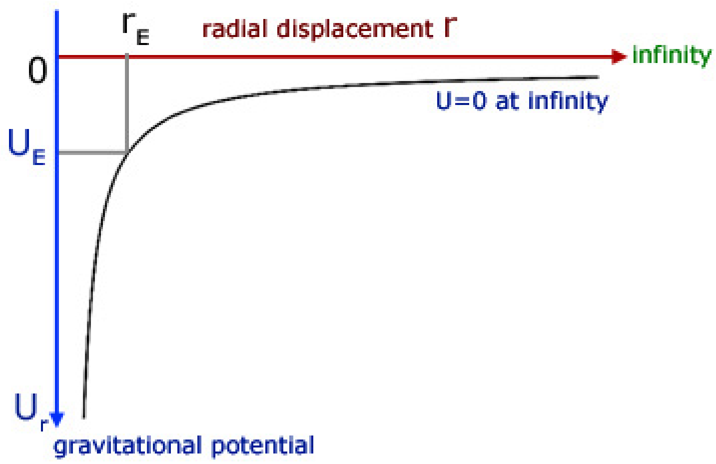
$$U_E = -W$$

substituting for ***W*** from above(remembering that ***m*** = 1kg),

$$U_E = -\frac{GM_E}{r_E}$$

Therefore in the general case, the potential U_r at a point a distance ***r*** away from a large mass ***M*** is given by:

$$U_r = -\frac{GM}{r}$$



The shape of the curve of U_r against ***r*** is of the type $y = -x^{-1}$ which is a reflection of the curve $y = x^{-1}$ about the x-axis.

Gravitational field strength - ***g***

The field strength ***g*** at a point in a gravitational field is defined as the force on unit mass at the point.

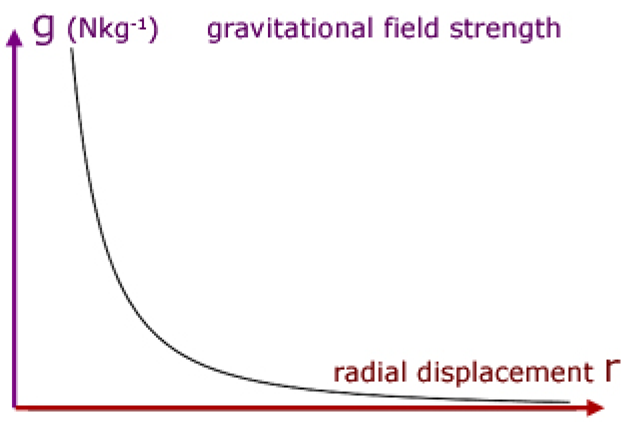
$$g = \frac{F}{m}$$

So the unit of gravitational field strength is Nkg^{-1} .

From Newton's Law of gravitation, the force on a mass ***m*** as a result of a mass ***M*** at a distance ***r*** is given by:

making the mass ***m*** unity (1 kg),

$$g = \frac{GM}{r^2}$$



The shape of the curve of ***g*** against ***r*** is of the type $y = x^{-2}$

Near the surface of the Earth, the value of ***g*** is considered to be constant at **9.8 Nkg^{-1}**.
The number should look familiar. This is also ***g*** , the acceleration due to gravity **9.8 ms^{-2}** .

The two ***g***'s are exactly the same, with the same dimensions.

$$N\ kg^{-1} = (kg\ ms^{-2})kg^{-1} = ms^{-2}$$

Derivation of the relation between ***g*** and ***U***

Consider a particle of mass ***m*** in a gravitational field.
In the absence of any applied force, the mass would be attracted to the major body producing the field.

Let the mass be held in position by a force ***F*** , acting in the opposite direction to the field direction.

Now if the force ***F*** moves the mass a very small distance ***δx*** against the field, the work done is given by:

(work = force x distance force moves)

$$\delta W = F\delta x$$

assuming that the force ***F*** is constant.

When the mass is static, the net force is zero. Forces are balanced. Since forces are vector quantities, the minus sign signifies opposite direction.

$$F = -mg$$

Substituting for ***F*** into our original equation,

$$\delta W = -mg\,\delta x \qquad \text{(i}$$

By definition, gravitational potential ***U*** is given by:

$$U = \frac{W}{m}$$

So ***δU***, the increase in ***U***, is given by:

$$\delta U = \frac{\delta W}{m}$$

Substituting for ***δW*** from equation (i ,

$$\delta U = \frac{-mg\,\delta x}{m}$$

cancelling the mass ***m*** gives,

$$\delta U = -g\,\delta x$$

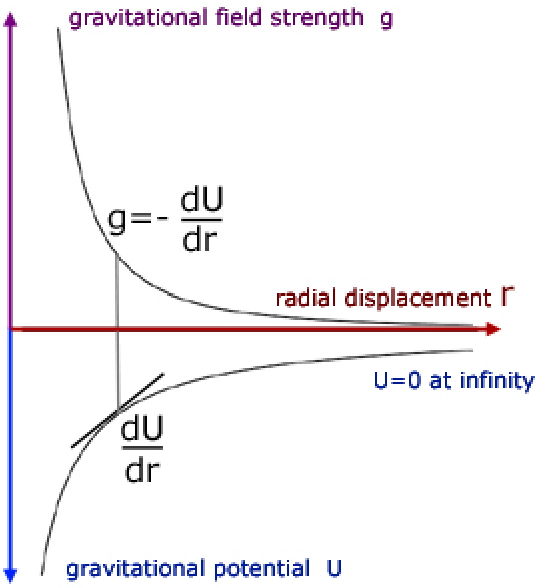
rearranging, to make ***-g*** the subject,

$$\frac{\delta U}{\delta x} = -g$$

In the limit, $\frac{\delta U}{\delta x} \rightarrow \frac{dU}{dx}$, therefore

$$\frac{dU}{dx} = -g$$

$$\underline{g = -\frac{dU}{dx}}$$



From the graph it can be seen that the gravitational field strength ***g*** at a radius ***r*** is equal to minus the value of the gradient of the gravitational potential ***U*** .

Note that ***g*** is positive because the value of dU/dr itself is negative. Multiplying a negative by a negative gives a positive.

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Energy in orbits

The energy E_r of a satellite of mass ***m*** in orbit, of radius ***r*** around a large body of mass ***M***, is the sum of the satellite's PE and KE respectively,

$$E_r = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

This equation can be simplified by eliminating v^2 .

Recalling the equation describing the circular motion of the satellite,

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

cancelling ***r***

$$mv^2 = \frac{GMm}{r}$$

and making v^2 the subject.

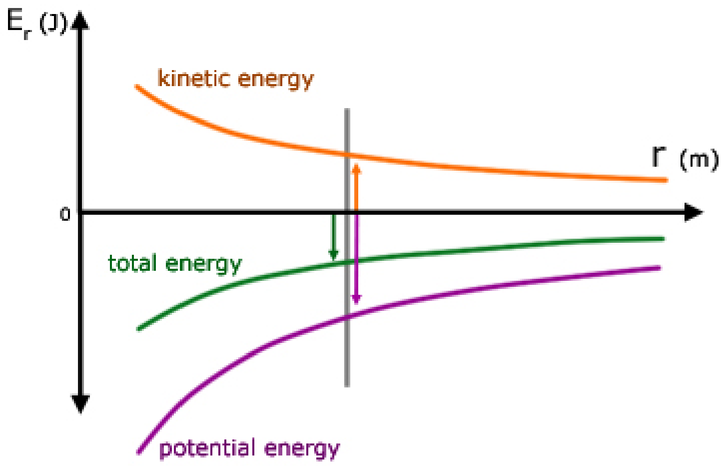
$$v^2 = \frac{GM}{r}$$

We can now substitute for v^2 in the initial energy equation:

$$E_r = -\frac{GMm}{r} + \frac{GMm}{2r}$$

So the total energy E_r of the satellite in its orbit is given by:

$$E_r = -\frac{GMm}{2r}$$



note:

1. the total energy of the satellite is always negative

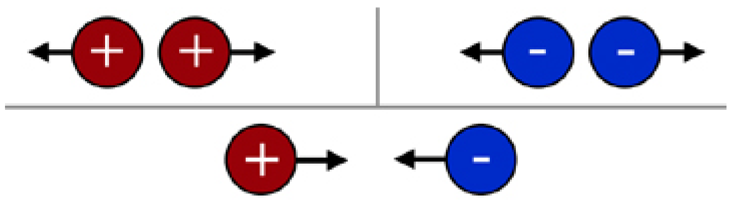
2. the PE component of the energy is twice as large in absolute terms as the KE component

Also, for any particular circular orbit with radius ***r*** , the individual values of kinetic and potential energies are constant.

By contrast, with elliptical orbits the values of potential and kinetic energies are not constant. They vary such that when one is large the other is small and vice versa. It must be remembered that the sum of potential and kinetic energies is always constant for a particular orbit.

Electric Fields - Part 1

Law of Electrostatics



Like charges repel, while unlike charges attract.

The convention for charge relates to early electrostatic experiments with particular materials.

glass rubbed with **silk** gives a **positive** charge

ebonite rubbed with **fur** gives a **negative** charge

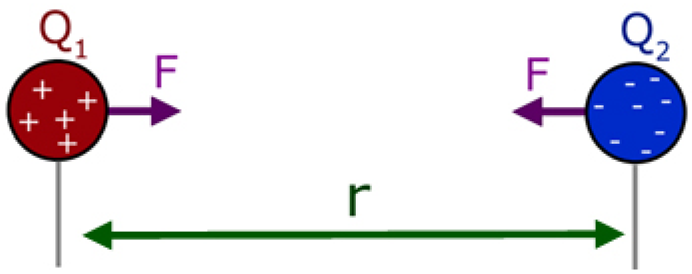
Using these results materials and particles are assigned charge:

perspex(positive), polythene(negative),

electrons(negative), protons(positive) etc.

The decision to make the charge from glass and silk positive, and the charge from ebonite and fur negative was completely arbitrary. The two charges could easily have been transposed, resulting in a positive charge on the electron!

Coulomb's Law



The force F between two point charges Q_1 & Q_2 is directly proportional to the product of the charges and inversely proportional to the square of the distance r between them.

$$F \propto Q_1 Q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{Q_1 Q_2}{r^2}$$

Making the proportionality into an equation by introducing a constant ***k*** :

$$F = k\frac{Q_1 Q_2}{r^2}$$

The value of ***k*** is given by :

$$k = \frac{1}{4\pi\varepsilon}$$

Hence the Coulomb's Law equation becomes :

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2}$$

The quantity ε is called the **permittivity**. It has a value depending on the medium surrounding the charges.

The symbol ε_o (epsilon nought) is used to describe the permittivity of free space, ie a vacuum.

The value of ε_o can be found from 'Maxwell's equations'. The result includes another constant μ_o (mu nought).

μ_o is called the **magnetic permeability** of free space.

The two are connected by the following relation, where ***c*** is the velocity of light :

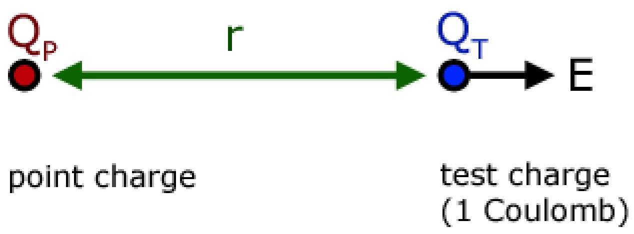
$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

The **unit of charge** is the **coulomb** (**C**). This is a S.I. derived unit.
By definition, a coulomb is the charge passed when a current of 1 ampere flows for 1 second.

$$\mathbf{1\ C = 1\ AS^{-1}}$$

Electric field strength E

The electric field strength at a point is equal to the force on a unit charge at the point.



From Coulomb's Law,

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_P Q_T}{r^2}$$

where,

Q_T is a unit test charge (1 C)

Q_P is any charge at a point

By definition, electric field strength is force/unit charge. So at the point where charge Q_T is positioned the field strength E is given by :

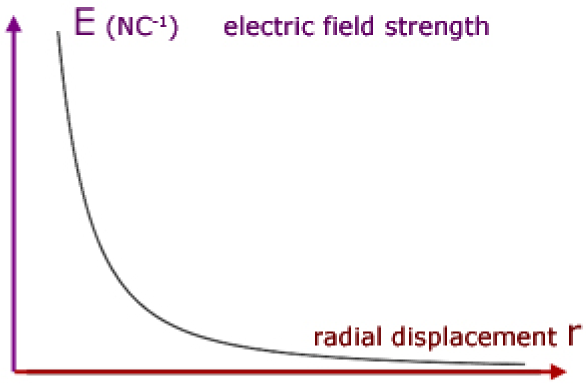
$$E = \frac{F}{Q_T}$$

But Q_T is a unit charge, therefore $E = F$.

Substituting for F in the initial Coulomb's Law equation,

$$E = \frac{1}{4\pi\varepsilon}\frac{Q_P}{r^2}$$

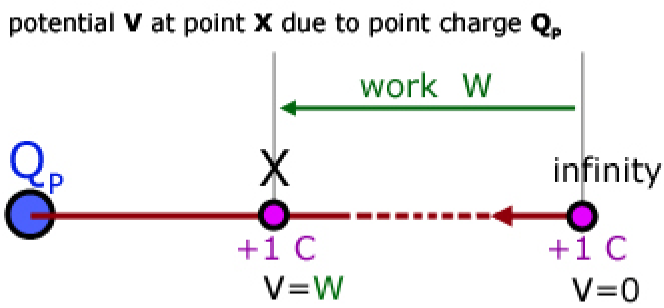
We can now see how electric field strength ***E*** varies with distance ***r*** from the point.



Electric potential V

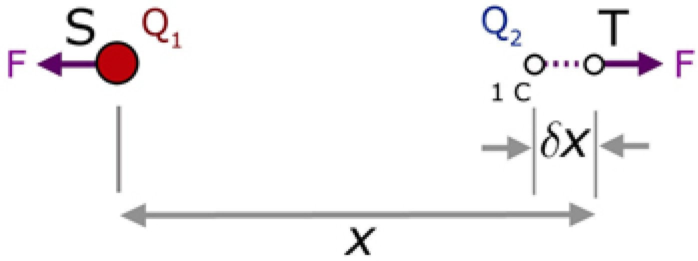
The electrical potential V at a point in an electric field is defined as:

being numerically equal to the work done ***W*** in transferring a unit positive charge from infinity to the point.



From Coulomb's law, the force between two point charges Q_1 and Q_2 is given by :

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2}$$



Q_1 is positive at point **S**. Q_2 is also positive but at point **T**.

Q_1 exerts a repulsive force F on Q_2.

Q_2 exerts a repulsive force F on Q_1 .

Consider an external force moving Q_2 at **T** an infinitesimal distance δx towards Q_1 at **S**.

Because the distance δx is so small, the repulsive force F may be considered to be constant during the movement.

Using,

work = force x distance force moves

the work done δW is given by :

$$\delta W = -F\delta x$$

The negative sign indicates that work is done against the field. That is, the motion is in the opposite direction to the direction of repulsion.

Substituting into this equation for F, from the Coulomb's Law equation (above) :

$$\delta W = -\frac{Q_1 Q_2}{4\pi\varepsilon x^2}\delta x$$

Therefore the total work W done in bringing the charge Q_2 from infinity to a point a distance r from **S** (where $x = r$) is given by:

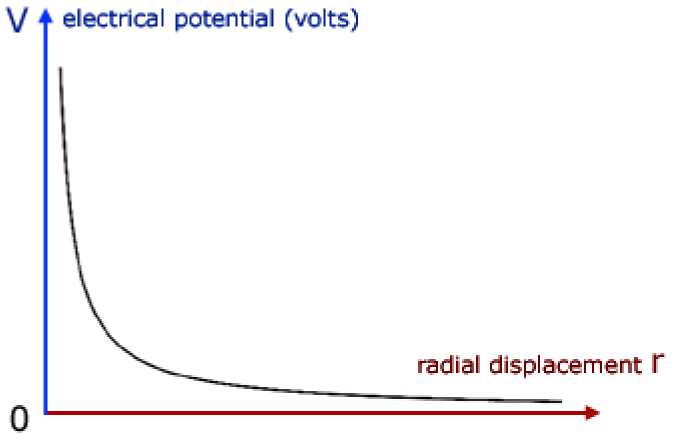
$$W = -\frac{Q_1 Q_2}{4\pi\varepsilon}\int_{\infty}^{r}\frac{1}{x^2}dx$$

integrating between the limits of ***r*** and infinity,

$$W = -\frac{Q_1 Q_2}{4\pi\varepsilon}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$W = \frac{Q_1 Q_2}{4\pi\varepsilon r}$$

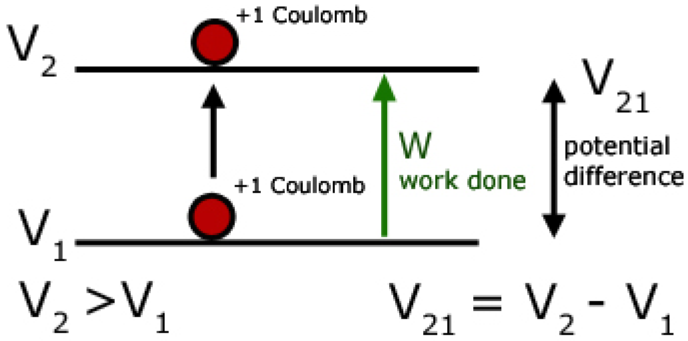
$$V = \frac{1}{4\pi\varepsilon}\frac{Q}{r}$$



The curve follows a simple inverse relation similar to y = 1/x .

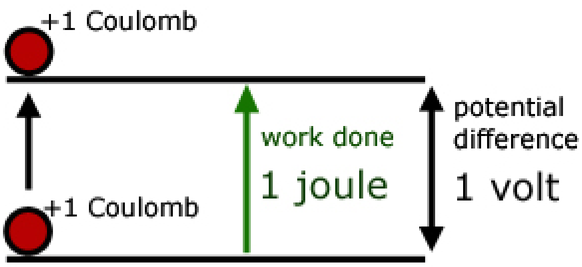
Electric potential difference

The potential difference between two points in an electric field is numerically equal to the work done in moving a unit + charge from the lower potential point to the higher.



From the definition of potential comes the definition of the volt, with specific units for work and charge.

The potential difference between two points is 1 **volt** if one joule of energy is used in moving 1 Coulomb of charge between the points.



Simply put, the number of volts is equal to the energy involved in moving 1 Coulomb of charge between points. So a 12V battery produces 12J of energy for every Coulomb moved between its terminals.

The equation connecting work ***W***, charge ***Q*** and potential difference ***V*** is as follows :

$$W = QV$$

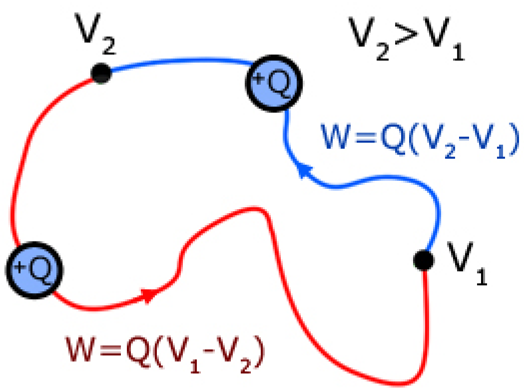
(joules = Coulombs x volts)

The diagram below illustrates that the work done in taking charge around a closed loop is zero.

Work **W** is done by the electric field in moving the charge from V_1 to V_2 .

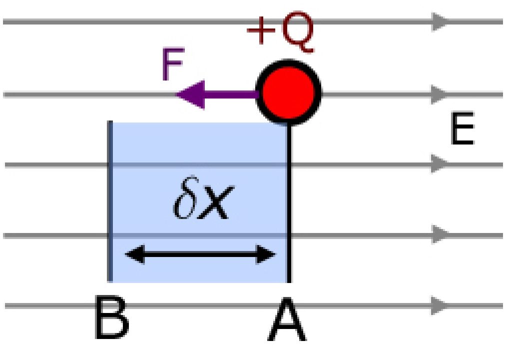
However, work **-W** must be done against the field to return the charge back to V_1 .

So the sum amount of work done = **W** + (**-W) = 0**



Relation between E and V

Consider a charge **+*Q*** being moved by a force ***F*** from an arbitrary point **A** to another point **B** against an electric field of strength ***E***.



The distance moved, ***δx*** , is very small, such that the force ***F***may be considered constant.

Hence the work done ***δW*** by the force is :

$$\delta W = F\delta x$$

The force is equal to the force exerted by the field on the charge, but in the opposite direction(note negative sign).

$$F = -EQ$$

Substituting in the original equation for ***F*** gives :

$$\delta W = -EQ\delta x$$

From the definition of potential difference, ***W = QV*** .

Therefore, if the potential difference between **A** & **B** is ***δV*** :
($V_B > V_A$)

$$\delta W = Q\delta V$$

Substituting for ***δW*** ,

$$-EQ\delta x = Q\delta V$$

cancelling the ***Q***'s and rearranging,

$$-E = \frac{\delta V}{\delta x}$$

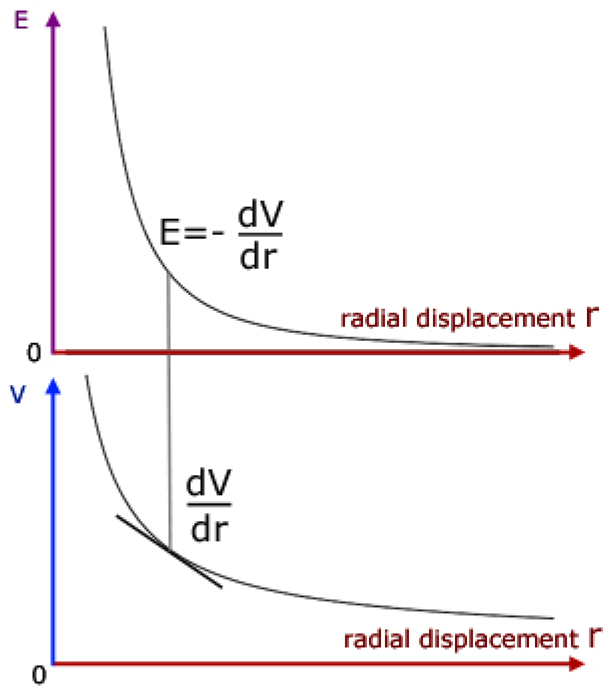
In the limit as ***δV*** and ***δx*** tend to zero,

$$\frac{\delta V}{\delta x} \rightarrow \frac{dV}{dx}$$

Multiplying both sides by -1 :

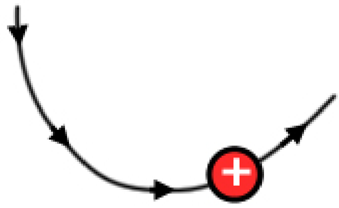
$$E = -\frac{dV}{dx}$$

The E-r and V-r graphs below show the relation clearly.
The gradient of the V-r graph is negative. So the negative of its gradient gives a positive value for E in the E-r graph.



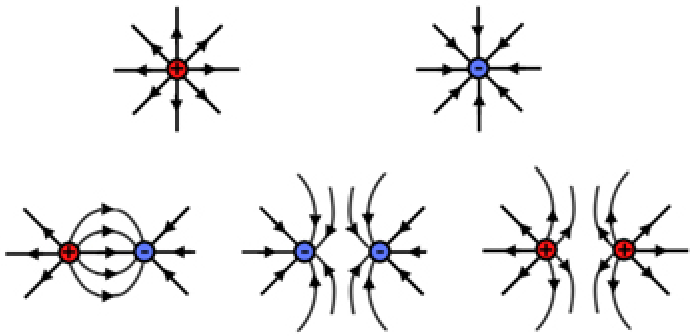
Electric Fields - Part 2

Electrostatic field diagrams



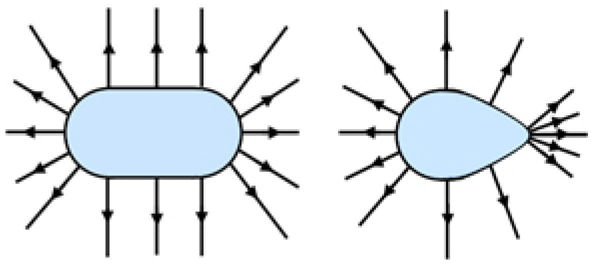
Electric field direction at a point is the direction of a small free moving positive charge if placed at the point.

Point charge electric field patterns:



Note the similarity of the patterns with single magnetic poles.

Common solid conductor shapes and their field patterns:

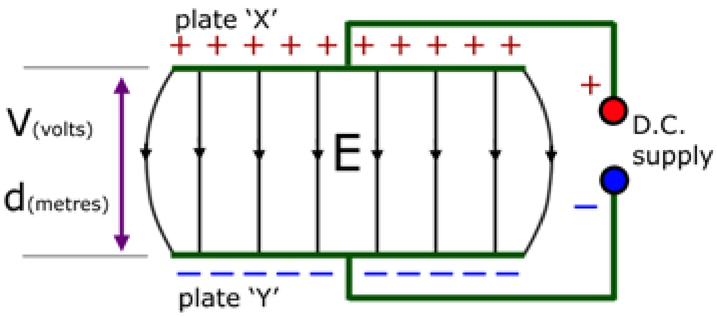


Observe how the field lines are not evenly distributed over each surface. The **charge density σ** (sigma) is the amount of charge per unit area. As might be expected, the field strength ***E*** at a point is directly proportional to **σ** (Gauss's theorem).

Note how ***E*** and **σ** are higher where the surfaces are highly curved.

<u>A uniform electric field</u>:

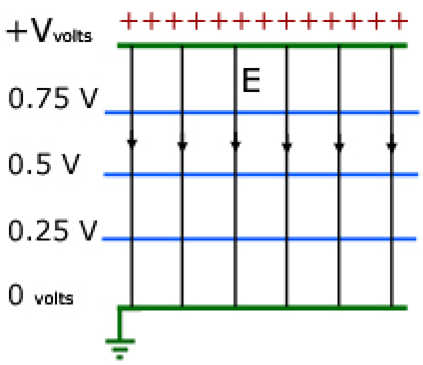
Two oppositely charged and parallel metal plates(**X** & **Y**) will produce a uniform electric field ***E*** between them. Note, at the edges the field lines are not evenly spaced. So the field there is not uniform.



The potential difference between the plates is constant along their length. The **potential gradient** (the drop in pd with distance) is the electric field strength ***E*** .
Since the pd ***V*** and the plate separation ***d*** are constant, ***E*** is also constant.

$$E = \frac{V}{d}$$

Lines of the same potential are called **equipotentials** . These are placed at right angles to the electric field lines. As a result of the field being uniform, the equipotential lines are equally spaced.

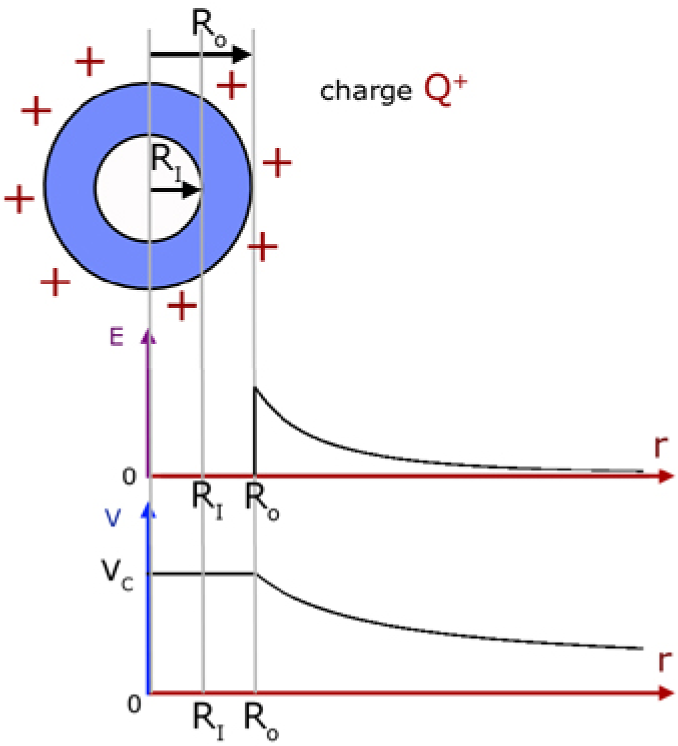


<u>Fields in and around hollow objects</u>

The simplest case is of a sphere.

outside of the sphere : ***E*** and ***V*** are the same as for a point charge at the sphere centre.

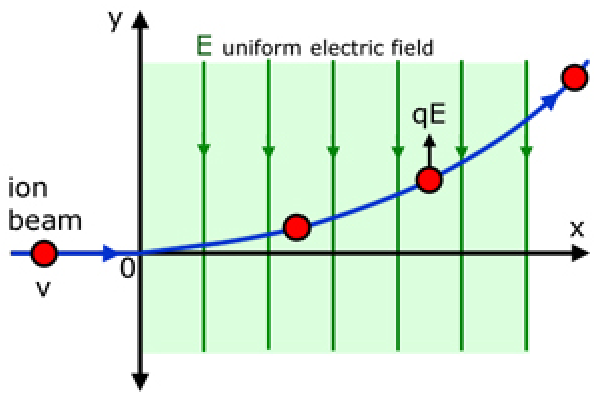
inside the sphere : ***E*** is zero in the material of the shell and in the enclosed space, while ***V*** is constant throughout (V_C).



Deflection of ion beams

From our definition of field strength (***E=F/Q***) , making the force ***F*** the subject of the equation, for a small charge ***q*** related to a particle :

$$F = qE$$



This force is constant at right angles to the original velocity ***v***.The vertical displacement $\boldsymbol{S_y}$ of the particle can be found from one of the 'equations of motion' used in mechanics :

$$s_y = ut + \tfrac{1}{2}at^2$$

where,

> ***u*** is the original velocity, in this case ***u*** = 0
> ***t*** is the period of acceleration
> ***a*** is the acceleration

Substituting for ***u*** ,

$$s_y = \tfrac{1}{2}at^2$$

From Newton's 2nd Law of Motion,

$$F = ma$$

Eliminating ***F*** , using our original expression for field strength

$$ma = qE$$

Rearranging to make the acceleration ***a*** the subject,

$$a = \frac{qE}{m}$$

Returning to the expression for vertical displacement and substituting for ***a*** ,

$$s_y = \tfrac{1}{2}\left(\frac{qE}{m}\right)t^2$$

The expression for horizontal displacement $\boldsymbol{S_x}$ can be rearranged to give an expression for ***t*** .

$$s_x = vt \qquad t = \frac{s_x}{v}$$

By substituting for ***t*** into the $\boldsymbol{S_y}$ equation we obtain our final expression :

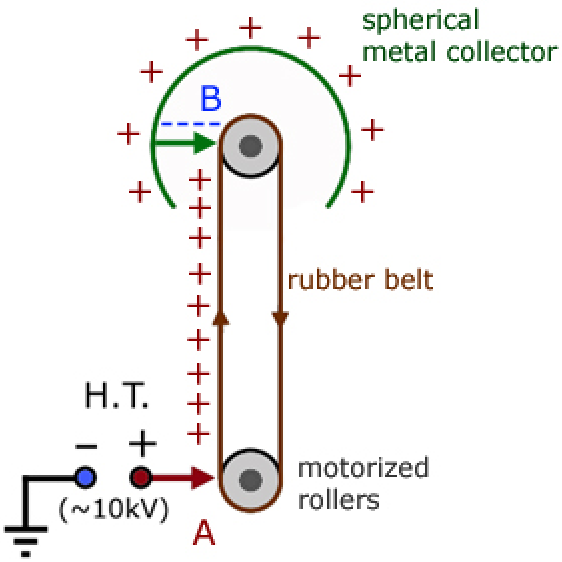
$$s_y = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{s_x}{v}\right)^2$$

$$s_y = \tfrac{1}{2}\left(\frac{qE}{mv^2}\right)s_x^{\,2}$$

The equation is of the form $\boldsymbol{y = kx^2}$, where k is a constant. The curve of the function is therefore a **parabola**.

Van de Graaf Generator

The Van de Graaf generator is a machine for producing extremely high potential differences (>10^7 V).



At the bottom (A), positive ionised air molecules are created at the points of a metal 'comb'. The molecules repel and are 'sprayed' onto the rubber belt.

The belt in turn takes the charges up to another comb (B). Here the charges induce negative charges which spread over the inside of the sphere. Positive charge is induced on the outside of the sphere.

At the same time, negative charges from the inside of the sphere are sprayed onto the belt to neutralise the positive ones. As the belt rotates more positive charge is taken up to the sphere and the cycle repeats itself.

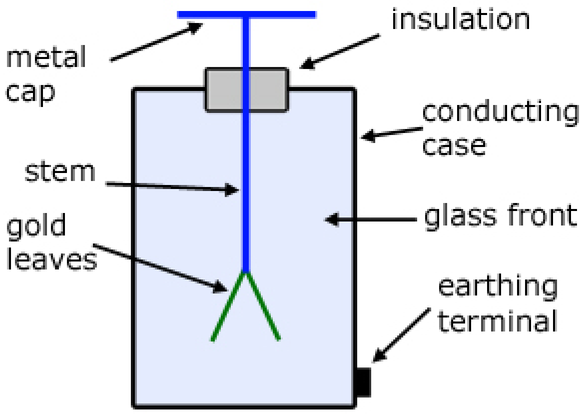
Gold Leaf Electroscope

The Gold Leaf Electroscope is a simple device but effective in the following areas:

measuring electrical potential

detecting the presence of charge close to it

determining whether a charge is positive or negative



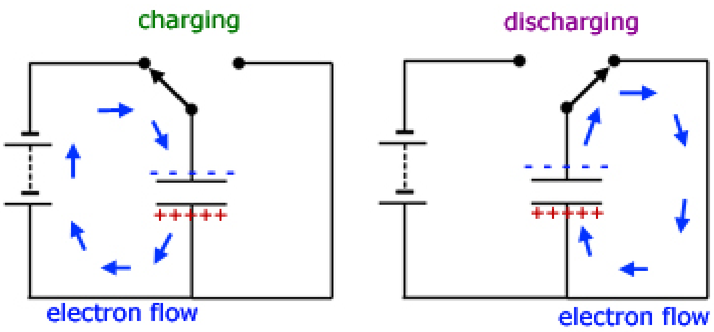
The two gold leaves are freely suspended and diverge when charged. The degree of divergence is a measure of the charge imparted.

Unfortunately space precludes a more detailed explanation of this amazing device at present.

Capacitors - Part 1

Action

Capacitors are electrical components used to store charge. Their construction is simply two equal area conducting plates, with an insultor(**dielectric**) sandwiched in between.



When the switch is turned to the left, there is an instantaneous flow of current. By the action of the battery electrons move in a clockwise sense. They are taken from the lower plate and deposited on the upper one.

In a very short time all motion ceases. The p.d. across the plates is now the same as that across the battery, but in the opposite direction. With the positive of the battery connected to the positive of the capacitor, no p.d. exists. So no current flows.

In this state the capacitor is said to be 'fully charged'. Charges on upper and lower plates are of opposite type and equal in quantity.

Capacitance

Capacitance is the measure of a capacitor to store charge. The larger the capacitor the more charge can be stored per volt of p.d. across the plates.

$$C = \frac{Q}{V}$$

where,

C is the capacitance in Farads (F)
Q is the charge in Coulombs (C)
V is the p.d. between the plates

The unit of capacitance is called the **Farad**.

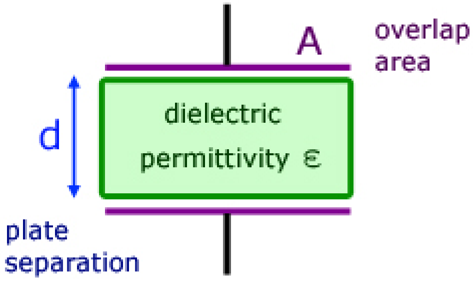
By definition, a capacitor has a capacitance of 1 Farad when 1 Coulomb of charge is stored with a p.d. of 1 volt across the plates.

Hence the units of Farads are Coulombs per volt (CV^{-1})

One Farad is too large a unit for ordinary circuits. Instead smaller derivative units are used, eg microfarads (μF) and picofarads (pF).

The Parallel Plate Capacitor

The capacitance of a parallel plate capacitor can easily be derived from first principles



Starting with our basic equation for capacitance,

$$C = \frac{Q}{V}$$ (i

The charge ***Q*** is equal to the charge density ***σ*** multiplied by the area ***A*** .

$$Q = \sigma A$$

Substituting for ***Q*** in the first equation (i ,

$$C = \frac{\sigma A}{V}$$ (ii

Results from **Gauss's Theorem*** give electric field strength ***E***in terms of charge density ***σ*** and permittivity ***ε*** :
* an advanced theory not dealt with here

$$E = \frac{\sigma}{\varepsilon}$$

Rearranging to make ***σ*** the subject,

$$\sigma = \varepsilon E$$

Now substituting for ***σ*** in equation (ii

$$C = \frac{\varepsilon E A}{V}$$ (iii

For the uniform field inside a capacitor,

$$E = \frac{V}{d}$$

hence,

$$V = Ed$$

Substituting for ***V*** into equation (iii

$$C = \frac{\varepsilon EA}{Ed}$$

$$\underline{C = \frac{\varepsilon A}{d}}$$

Note: the expression for electric field strength ***E*** from Gauss's law is for an infinite plate area ***A*** . This result is therefore an approximation.

Relative Permittivity

The definition of relative permittivity is the ratio of the capacitance of a capacitor with a dielectric to that of a capacitor without (ie free space).

$$\varepsilon_r = \frac{C}{C_o}$$

By looking at capacitance we can obtain an expression linking relative permittivity ε_r , the permittivity of free space ε_o and the permittivity of the dielectric being used.

$$C = \frac{\varepsilon A}{d} \qquad C_o = \frac{\varepsilon_o A}{d}$$

We find ε_r by dividing the first equation by the second :

$$\frac{C}{C_o} = \left(\frac{\varepsilon A}{d}\right)\left(\frac{d}{\varepsilon_o A}\right) = \frac{\varepsilon}{\varepsilon_o}$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$$

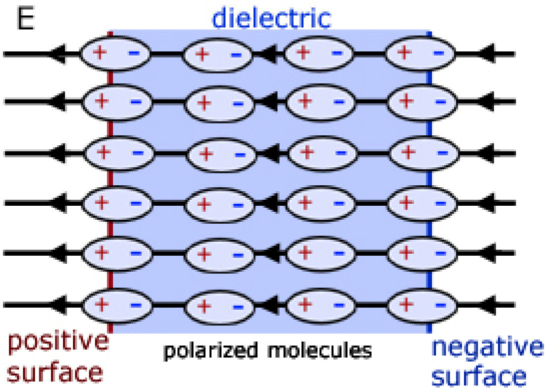
$$\underline{\varepsilon = \varepsilon_o \varepsilon_r}$$

Note: because ε_r is a ratio of permittivities, it has no units

Dielectrics

To understand the action of dielectric materials it is important to appreciate what is happening on the molecular level.

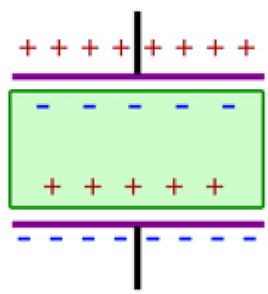
When an electric field is applied to a dielectric the positioning of the components of atoms/molecules is changed.



Positive atomic nuclei are moved to their limit a short distance and point towards the negative plate. Electron clouds around atoms become misshapen with the bulk of their charge pointing towards the positive plate. In this way atoms/molecules become polarized, with opposite charges tending to be concentrated at either end.
The result is that the surfaces of the dielectric facing the capacitor's plates become charged. A positive plate opposes the negative face of the dielectric, while a negative plate opposes the dielectric's positive face.

A dielectric between the plates of a capacitor modifies capacitance in two particular circumstances:

An isolated charged capacitor

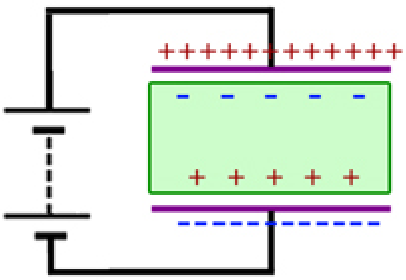


The negative charge on the top surface of the dielectric combines with the positive charge on the top plate to lessen the overall potential in the area. (The area is less positive.)

The positive charge on the bottom surface of the dielectric combines with the negative charge on the bottom plate to increase the potential there. (The area is less negative.)

The result is that there is less potential difference (***V***) across the capacitor. Since ***Q* = *CV***, ***Q*** is unchanged and ***V*** decreases, then ***C*** increases.

A capacitor in-circuit with a battery



The p.d. ***V*** across the plates is maintained by the battery. The surface charges on the dielectric cause more electrons to be drawn from the positive plate and be deposited on the negative plate.
So the overall charge ***Q*** on the plates is increased. Since ***V*** is constant and ***Q* = *CV*** then ***C*** increases.

For different reasons, the effect of introducing a dielectric substance between the plates of a capacitor has the effect of increasing the capacitance.

Below is a table of common relative permittivities.

Note how the relative permittivity for water is much higher than the rest. This is because water(along with many other liquids), has **polar molecules** . These are polarized. That is they have + and - ends. without any field being applied. Polar molecules in liquids readily align themselves to an electric field. In this way more surface charges are produced and the effect is greater.

material	rel. permittivity (ε_r)
vacuum	1.0
air	1.00058986 (STP)
ebonite	3
glass	5
mica	7
paper	3.85
polythene	2.25
polystyrene	2.4 - 2.7
rubber	7
water	80.1 (room temp.)

Capacitors - Part 2

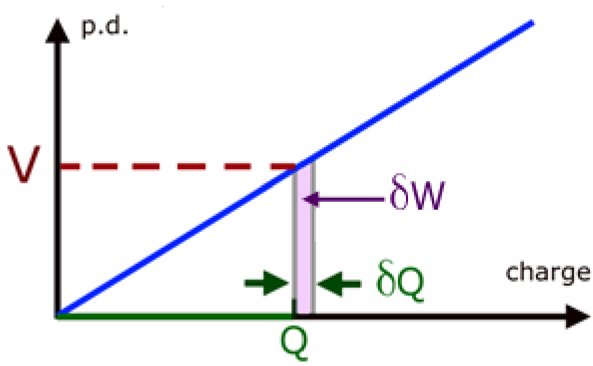
Energy stored in a capacitor

The energy stored in a capacitor is in the form of electrical potential energy. This is has two components:

work done in adding electrons to the negative plate

work done in removing electrons from the positive plate

Consider a *partially* charged capacitor, with a p.d. of ***V*** volts across it and charge ***Q*** on it plates.



Now during charging, consider a small charge ***δQ*** moving from one plate to another. If ***δQ*** is very small then the increase in p.d. is also very small. So ***V*** may be considered approximately the same.

Hence the work done ***δW*** , is given by :

$$\delta W = V\delta Q$$

Recalling the equation for capacitance ***C*** and rearranging to make ***V*** the subject :

$$C = \frac{Q}{V} \qquad V = \frac{Q}{C}$$

Substituting for ***V*** in our original equation.

$$\delta W = \frac{Q}{C}\delta Q$$

The total work done is the area under the curve for 0 to Q_o, where Q_o is the maximum charge stored.

$$W = \int_0^{Q_o} \frac{Q}{C} dQ$$

Integrating between the limits gives the result :

$$W = \frac{Q_o^2}{2C}$$

In the general case, we can write $Q = Q_o$.

$$W = \frac{Q^2}{2C}$$

Recalling the capacitor equation again and making Q the subject :

$$C = \frac{Q}{V} \qquad Q = VC$$

Substituting for Q into the equation for W :

$$W = \frac{(VC)^2}{2C} = \frac{V^2C^2}{2C} = \frac{V^2C}{2}$$

$$W = \tfrac{1}{2}CV^2$$

Now substituting for $CV = VC = Q$,

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(CV)V$$

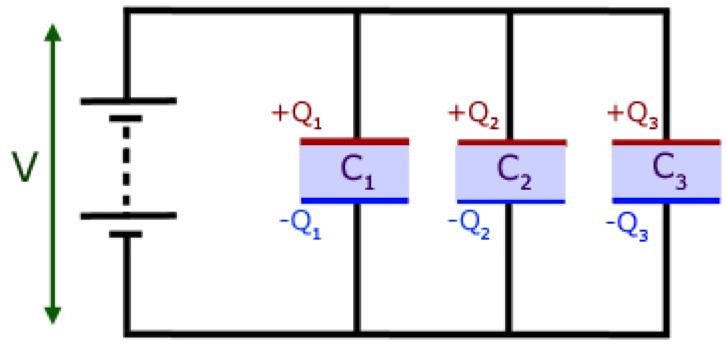
$$W = \tfrac{1}{2}QV$$

Summarizing,

$$\underline{W = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV}$$

Capacitors in parallel

Capacitors in parallel have the same p.d. across them.



Writing ***Q = CV*** for each capacitor and adding :

$$Q_1 = C_1V$$
$$Q_2 = C_2V$$
$$Q_3 = C_3V$$
$$Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

Putting Q_T as the total charge,

$$Q_T = Q_1 + Q_2 + Q$$

Hence,

$$Q_T = C_1V + C_2V + C_3V$$

Factorising,

$$Q_T = (C_1 + C_2 + C_3)V$$

But

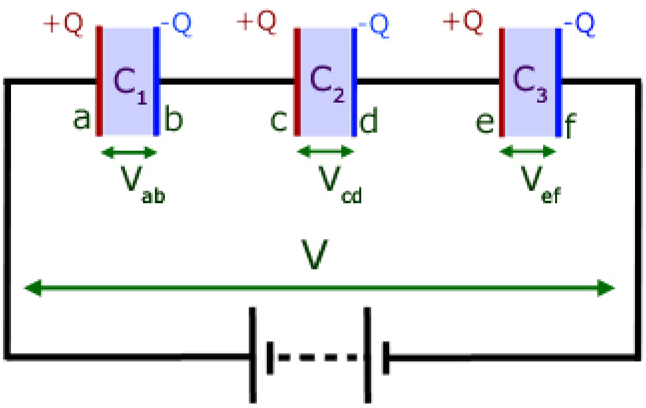
$$Q_T = CV$$

Therefore,

$$\underline{C = C_1 + C_2 + C_3}$$

Capacitors in series

The battery removes charge **Q**$^-$ from plate '**a**' and deposits it on plate '**f**'. Plate 'a' is therefore left with a charge **Q**$^+$ on its plates.
Each charged plate then induces an opposite charge in its opposing plate.
The central capacitor $\mathbf{C_2}$ has a positive charge on plate 'c' because electrons are removed from it to make plate 'b' negative. Plate 'd' is made negative by induction with 'c'.



Making the p.d. ***V*** the subject for each capacitor and adding:

$$V_{ab} = \frac{Q}{C_1}$$

$$V_{cd} = \frac{Q}{C_2}$$

$$V_{ef} = \frac{Q}{C_3}$$

$$V_{ab} + V_{cd} + V_{ef} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V_{ab} + V_{cd} + V_{ef} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)Q$$

Summing the p.d. around the circuit, the supply p.d. is ***V*** is given by:

$$V = V_{ab} + V_{cd} + V_{ef}$$

Hence

$$V = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)Q$$

Recalling that,

$$V = \frac{Q}{C}$$

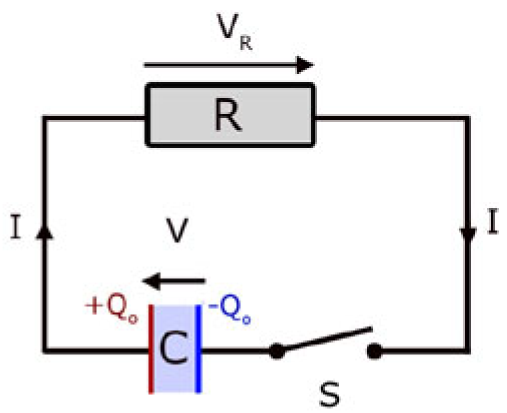
By similarity, it follows that :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Charging & discharging a capacitor through a resistor

When the switch is closed charge from the capacitor flows through the resistor. The resistance R has the effect of limiting this flow.

For a fully charged capacitor of capacitance **C** let the p.d. be$\mathbf{V_o}$ and the charge $\mathbf{Q_o}$.



Consider the p.d. around the circuit at a time ***t*** seconds from the start of the discharge. Since there is no net p.d. in the circuit, by Kirchoff's 2nd law (relating to p.d. in a circuit) :

$$0 = V + V_R$$

Quoting the capacitor equation, with ***V*** the subject,

$$V = \frac{Q}{C}$$

Substituting for ***V*** in our initial equation,

$$0 = \frac{Q}{C} + V_R$$ (i

The current ***I*** is defined as the rate of charge of charge with time,

$$I = \frac{dQ}{dt}$$

Remember the Ohm's Law equation,

$$V = IR$$

Now, substituting for ***V*** and ***I*** into equation (i ,

$$0 = \frac{Q}{C} + \frac{dQ}{dt}R$$

Separating the differential operators ***dQ*** and ***dt*** ,

$$\frac{dQ}{Q} = -\frac{dt}{CR}$$

Since ***Q*** = ***Q_o*** when ***t*** = 0 and ***Q*** = ***Q*** when ***t*** = ***t*** , integrating between these limits :

$$\int_{Q_o}^{Q} \frac{dQ}{Q} = -\frac{1}{CR}\int_0^t dt$$

Hence,

$$\left[\log_e Q\right]_{Q_o}^{Q} = -\frac{1}{CR}\left[t\right]_o^t$$

Therefore,

$$\log_e Q - \log_e Q_o = -\frac{t}{CR}$$

and,

$$\log_e\left(\frac{Q}{Q_o}\right) = -\frac{t}{CR}$$

Changing into exponential form :

$$\frac{Q}{Q_o} = e^{\frac{-t}{CR}}$$

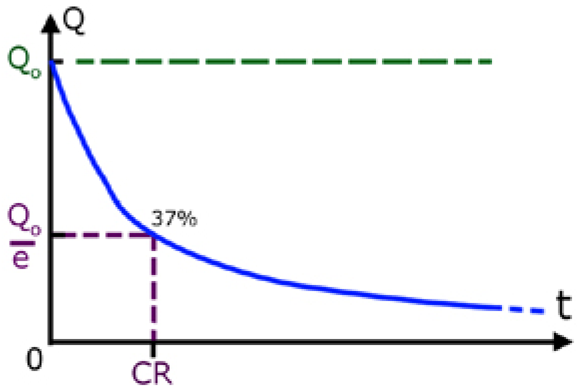
Rearranging into a more familiar form,

$$\underline{Q = Q_o e^{\frac{-t}{CR}}}$$

Substituting for $\boldsymbol{Q} = \boldsymbol{VC}$ and $\boldsymbol{Q_o} = \boldsymbol{V_o C}$, cancelling the ***C***'s

$$\underline{V = V_o e^{\frac{-t}{CR}}}$$

So both the charge and the p.d. decrease at an exponetial rate.



When the elapsed time $\boldsymbol{t} = \boldsymbol{CR}$ the charge remaining is approx. 37% of the original amount.

$$Q = Q_o e^{-1} = \frac{Q_o}{e} = 0.368Q_o$$

The p.d. across the resistor $\boldsymbol{V_R}$ and the current $\boldsymbol{I}$ through it are given by:

$$V_R = -V_o e^{\frac{-t}{CR}}$$

Substituting for $\boldsymbol{V_R}$, (using $\boldsymbol{V_R = IR}$) and making $\boldsymbol{I}$ the subject,

$$I = -\left(\frac{V_o}{R}\right)e^{\frac{-t}{CR}}$$

Note the minus signs in these equations.
This is a consequence of Kirchoff's 2nd law. Remember how the p.d. across the resistor and the capacitor are related.

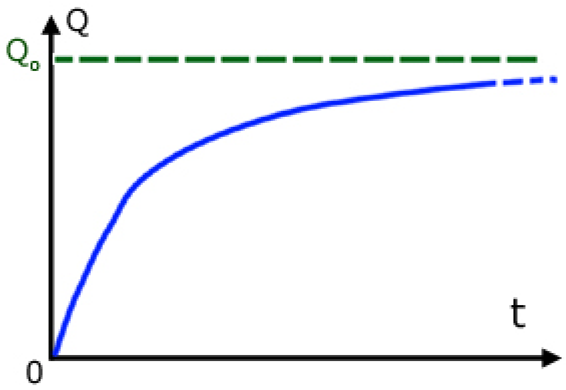
$$0 = V + V_R$$

If $\boldsymbol{V_R}$ is taken to the other side of the equation it becomes negative. Hence one p.d. is the negative of the other.

The second equation ($\boldsymbol{I}$) is obtained from the first by substituting $\boldsymbol{V_R = IR}$ and rearranging.

The curve of ***Q*** vs ***t*** for charging is :

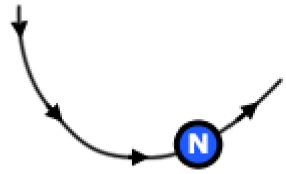
$$Q = Q_o\left(1 - e^{\frac{-t}{CR}}\right)$$



Unfortunately lack of space precludes this derivation.

Magnetic Fields - Part 1

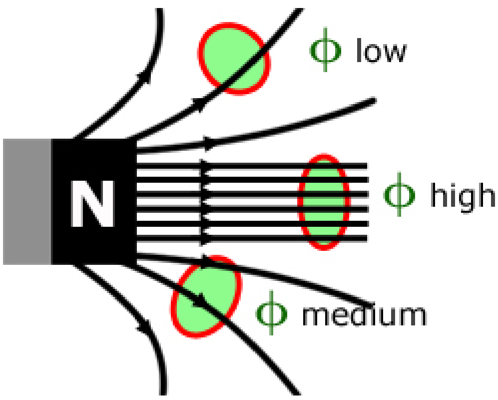
Fast revision: **Magnetic field lines** follow the direction of a free moving North Pole.



Magnetic Flux φ (phi)

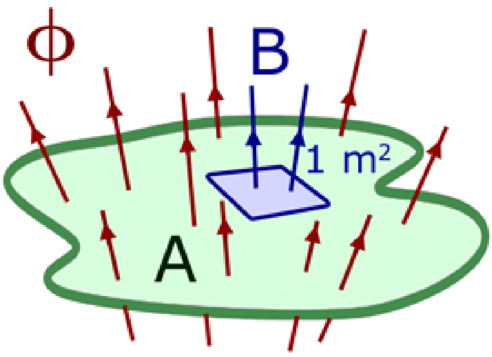
By definition, magnetic flux is a measure of the strength of a magnetic field over a given area perpendicular to it.

The diagram below shows how the magnetic flux **φ** over an area ***A*** varies around the pole of a magnet.



Flux Density B

We can refine the idea of flux by making the area unity ($1m^2$). This introduces a new concept - **magnetic flux density *B*** .



For *normal* area (area at right angles),

total magnetic flux = flux density x area

$$\phi = BA \qquad B = \frac{\phi}{A}$$

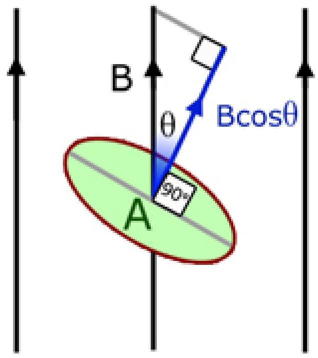
Units

The unit of flux is the **Weber** (**Wb**) and the unit of flux density is the **Tesla** (**T**).

A flux density of 1 Tesla is 1 Weber per square metre.

$$\mathbf{1\ T = 1\ Wbm^{-2}}$$

For an area ***A*** at an angle **θ** to the magnetic field, normal flux density has magnitude **Bcosθ** .

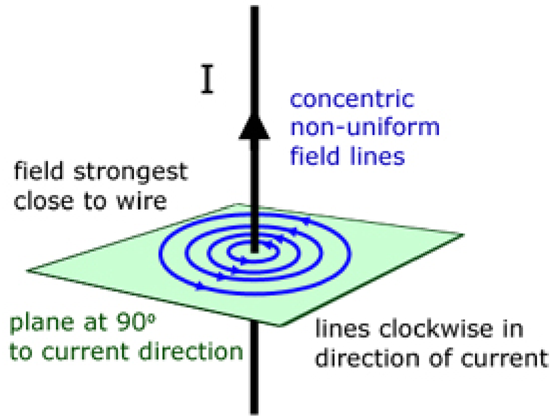


So the total normal flux over an area ***A*** at an angle **θ** to the field is given by :

$$\phi = AB\cos\theta$$

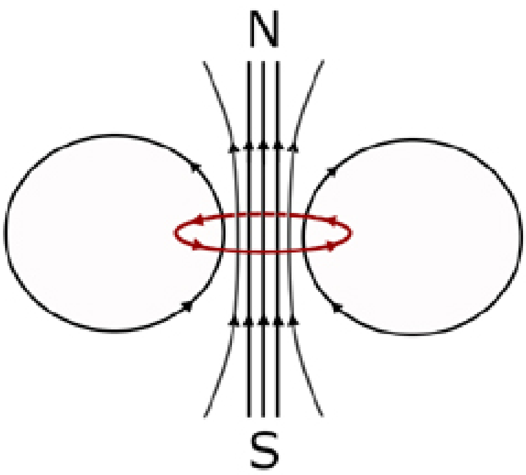
Magnetic fields around current-carrying conductors

The magnetic field around a current-carrying wire is a series of concentric field lines. The field is not uniform. The lines are not evenly spaced. The field is non-uniform, with lines tightly packed close to the wire and widely spaced away from it.

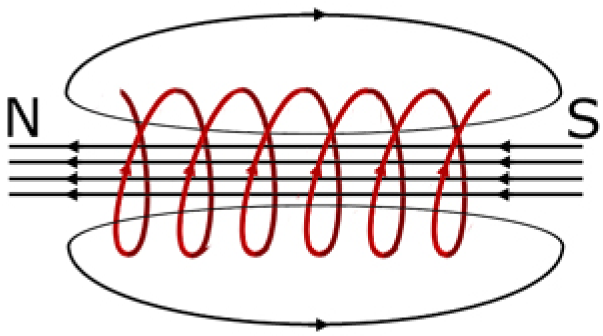


The direction of the lines of force is clockwise in the direction of the current direction.

The field around a plane circular coil resembles the field around a **short** bar magnet.

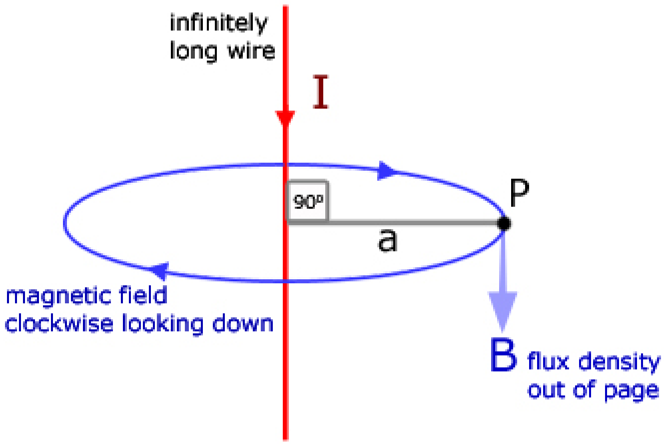


The field around a solenoid resembles the field around a **long** bar magnet.



Flux density for a straight wire

The diagram below illustrates the flux density ***B*** at a point **P** a distance ***a*** away from the wire.



The magnetic flux density ***B*** is described by the equation :

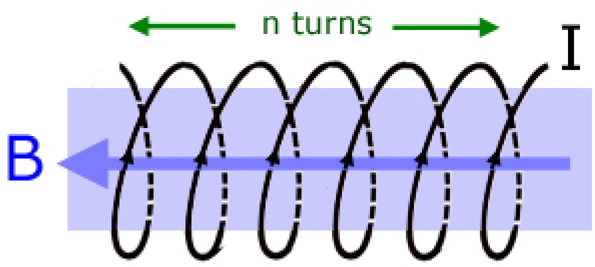
$$B = \frac{\mu_o I}{2\pi a}$$

where μ_o is the permeability of free space.

Unfortunately a full derivation of this equation cannot be given at present.

Flux density for an infinitely long solenoid

The diagram illustrates the flux density ***B*** in a solenoid with ***n***turns and coil current ***I***.



The magnetic flux density ***B*** is described by the equation :

$$B = \mu_o n I$$

where μ_o is the permeability of free space and ***n*** is the number of turns per unit length of the solenoid.

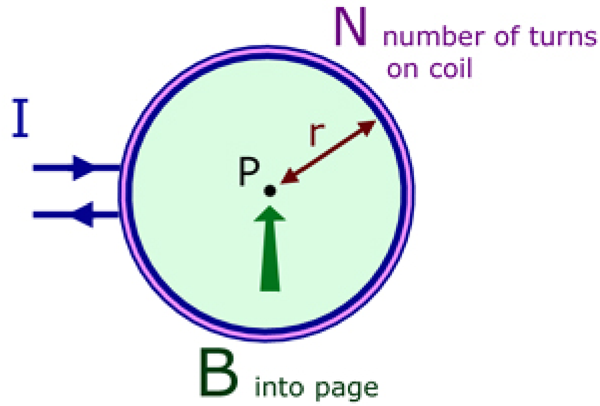
The value of ***B*** approximates to that of a real solenoid provided the solenoid's length is at least x10 its diameter.

The quantity ***nI*** is of significance. It is equal to the **magnetic field strength *H*** , with units of amp-turns/metre (Am^{-2}) .
note: *turns* ***n*** has no units

$$H = nI$$

Uniform magnetic fields - Helmholtz Coils

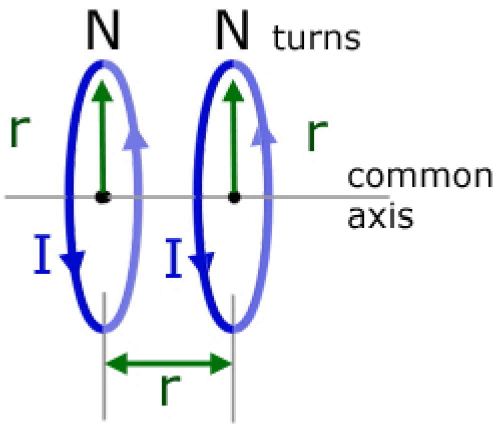
A single plane coil of radius ***r*** , turns ***N*** and current ***I*** produces magnetic flux density ***B*** at its **centre**.



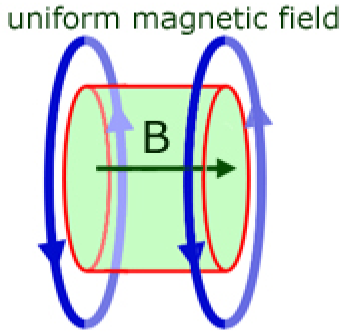
$$B = \frac{\mu_o NI}{2r}$$

where μ_o is the permeability of free space.

Helmholtz Coils produce a region of uniform magnetic field within a discrete volume. Two identical plane coils are aligned along a common axis and positioned a distance ***r*** apart, where ***r*** is the coil radius.



The current ***I*** passing through each coil is the same and in the same direction.



The magnetic flux density ***B*** in the volume of uniform field (shaded green) is given by :

$$B \approx 0.72 \frac{\mu_o NI}{r}$$

where μ_o is the permeability of free space.

Helmholtz coils are particularly useful for deflecting electron/ion beams. All charged particles follow a circular path when injected into a magnetic field at right angles to their motion. By measuring the radius of a path and whether the path is clockwise or anticlockwise, important information can be gleaned on the charge of a particle and its mass.

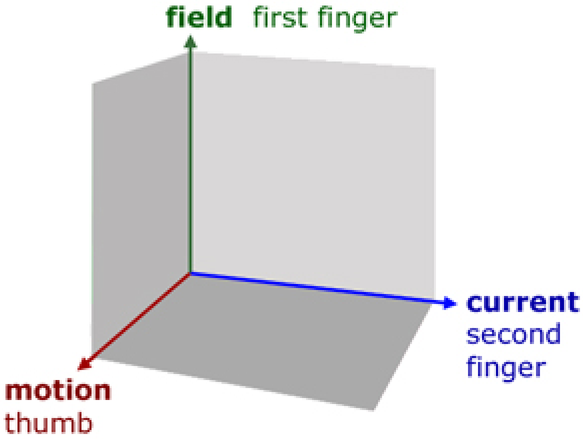
This method is particularly important in distinguishing ***α*** , ***β*** and ***γ*** particles from each other.

Magnetic Fields - Part 2

Flemming's Left Hand Rule

The rule describes the resulting directional motion of a current carrying conductor(or a moving charged positive particle) from the current and field directions.

The three quantities FIELD, CURRENT AND MOTION are mutually at right angles to each other.



using the left hand, position the first finger, second finger and thumb to form the x,y,z axes. The highlighted letters within the words help you remember the three quantities

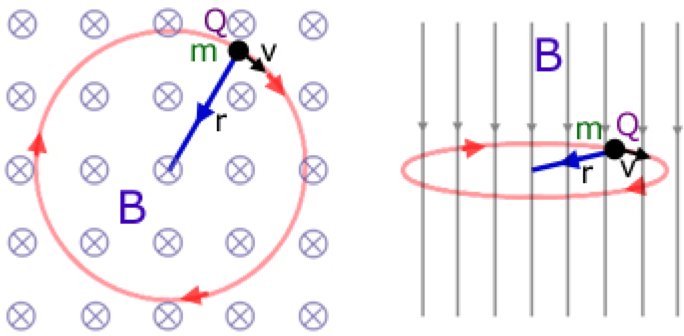
First finger - **F**ield direction

se**C**ond finger - **C**urrent direction

thu**M**b - **M**otion produced

Force on a charged particle in a magnetic field

The force on a charged particle moving in a magnetic field is at right angles to the velocity. This causes the particle to perform circular motion.



For a velocity at right angles to the field, the magnitude of the force ***F*** is simply the product of the magnetic flux density ***B***, the charge ***Q*** on the particle and its velocity ***v***.

$$F = BQv$$

Equating this force with the centripetal force, we obtain the relation :

$$\frac{mv^2}{r} = BQv$$

Rearranging to make the radius of motion ***r*** the subject :

$$\frac{mv}{r} = BQ \qquad r = \frac{mv}{BQ}$$

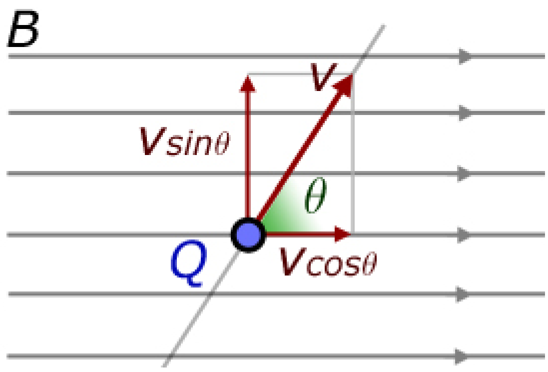
This is quite an important result.

The radius ***r*** is :

i) directly proportional to the momentum (***mv***)
ii) inversely proportional to the flux density ***B***
iii) inversely proportional to the charge ***Q***

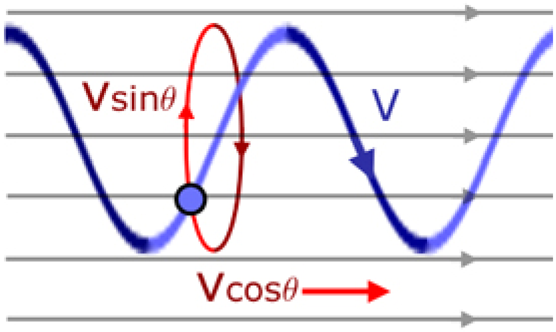
Whether a particle orbits clockwise or anticlockwise indicates charge type(+ or -). So with information from orbital radius particles can be identified in nuclear physics experiments.

Now consider a particle moving at velocity ***v*** at an angle ***θ*** to a magnetic field of flux density ***B***.



The velocity ***v*** is resolved along the field (***vsinθ***) and at right angles to it (***vcosθ***).

This means that the particle performs circular motion with a velocity at right angles to the circle. The result is a corkscrew motion.



Circular motion velocity is now ***vsinθ*** (not ***v***). Therefore the original equation for normal field and velocity is amended to :

$$F = BQv\sin\theta$$

Force on a metal conductor in a magnetic field

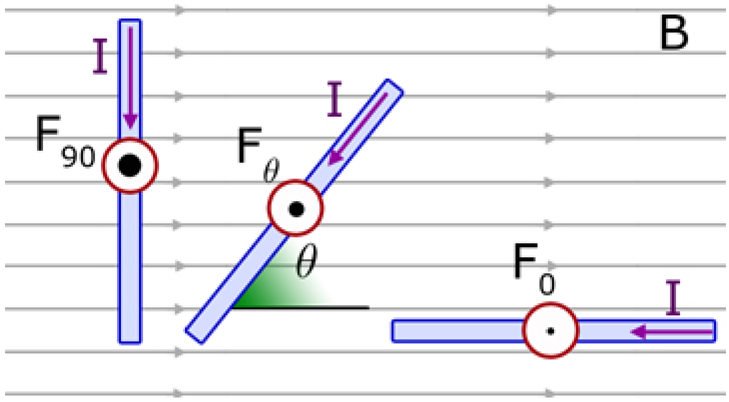
For a metal conductor of length ***L*** , carrying a current ***I*** , at right angles to a magnetic field of flux density ***B*** , the force ***F***upwards on the conductor is simply : (left-hand diagram)

$$F = BIL$$

For a metal conductor at an angle θ to the field, the force upwards on the conductor is : (centre diagram)

$$F = BIL\sin\theta$$

For a metal conductor in-line with the field (right-hand diagram) $\theta = 0^{o}$ and sine $0^{o} = 0$. Hence the force F is zero.



The expression $F = BIL\sin\theta$ can be derived from first principles.

Consider a metal conductor of length L with electrons travelling along it with an average drift velocity v.
The average time t for an electron to traverse the length L, (using time = distance/velocity) is :

$$t = \frac{L}{v}$$

If Q is the charge transferred along the conductor in time t , then the current I (by definition the charge passes in unit time) is :

$$I = \frac{Q}{t}$$

The charge Q is a particular number n of electrons :

$$Q = Ne$$

Substituting for Q and t into the expression for current I :

$$I = \frac{Ne}{\left(\frac{L}{v}\right)} \qquad I = \frac{Nev}{L} \qquad Nev = IL$$

Quoting the equation for a moving particle at an angle to a magnetic field :

$$F = BQv\sin\theta$$

Substituting for ***Q* :** ($Q = Ne$)

$$F = B(Ne)v\sin\theta$$

rearranging,

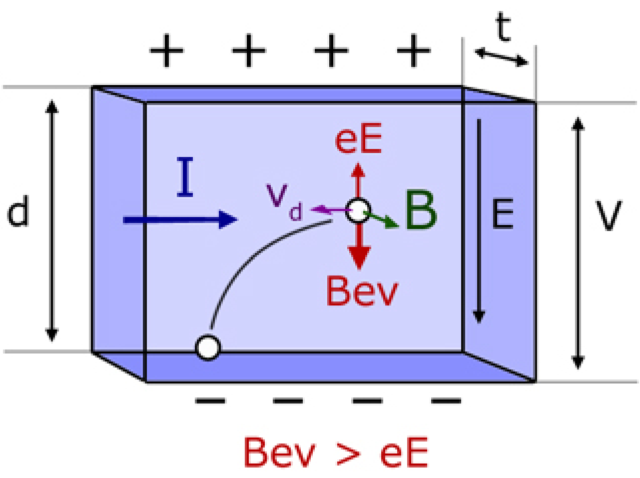
$$F = B(Nev)\sin\theta$$

Substituting for ***Nev*** from above,

$$\underline{F = BIL\sin\theta}$$

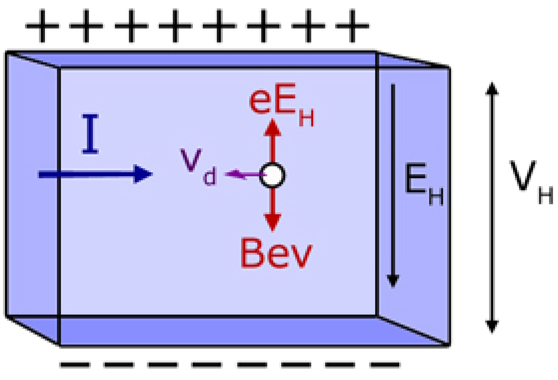
The Hall Effect

The Hall Effect refers to the potential difference that builds up across opposing faces of a cuboidal shaped material when it is carrying current and placed in a uniform magnetic field.



The potential difference ***V*** is a consequence of free electrons in the material being deflected by the magnetic field. The electrons move in arcs and build up on the bottom face. Removal of free electrons from the top half of the solid makes the top face positive.

The electric field ***E*** produced by the deposited electrons is very weak to start with. As more electrons accumulate, it grows stronger. The resultant force on electrons(Bev - eE) becomes weaker and weaker. Eventually a point is reached when the magnetic and electric forces are equal(Bev = eE) and no more electrons are added.



$$Bev = eE_H$$

When this happens the potential difference across the top and bottom faces is a maximum (***V*$_H$**). The p.d. is called the Hall Voltage and is a measure of the magnetic field passing through the solid.

An expression for the Hall Voltage can be obtained from a consideration of the forces on an electron, when they are balanced.

The downward magnetic force F is given by :

$$F = Bev$$

The upward electric force is given by :

$$F = eE$$

For a uniform electric field,

$$E = \frac{V_H}{d}$$

where ***d*** is the thickness of the sample.

Substituting for ***E*** into the electric force equation,

$$F = \frac{eV_H}{d}$$

Equating the magnetic force with the electric force :

$$Bev = \frac{eV_H}{d}$$

cancelling the **e**'s and making ***V*$_H$** the subject of the equation,

$$V_H = Bvd$$

Quoting the **Drift Velocity** equation from Electricity(conduction in metals),

$$I = nAve$$

Rearranging to make the drift velocity ***v*** the subject,

$$v = \frac{I}{nAe}$$

Now, substituting for ***v*** into the equation for V_H ,

$$V_H = \frac{BdI}{nAe}$$

The cross-sectional area **A** of the sample between left and right faces is simply the product of the height ***d*** and its thickness ***t***.

$$A = dt$$

Substituting for ***A*** in the new expression for V_H ,

$$V_H = \frac{BdI}{ndte} = \frac{BI}{nte}$$

Shuffling around bottom terms to make the equation more memorable :

$$\underline{V_H = \frac{BI}{net}}$$

The Hall voltage is larger in semiconductors than metals. This is because V_H is inversely proportional to the number of charge carriers/unit volume ***n***. Values for ***n*** are much smaller in semiconductors. So V_H is larger.

Typical values of V_H for specimens under similar conditions :

metals ~ 10^{-6} V

semiconductors ~ 10^{-2} V

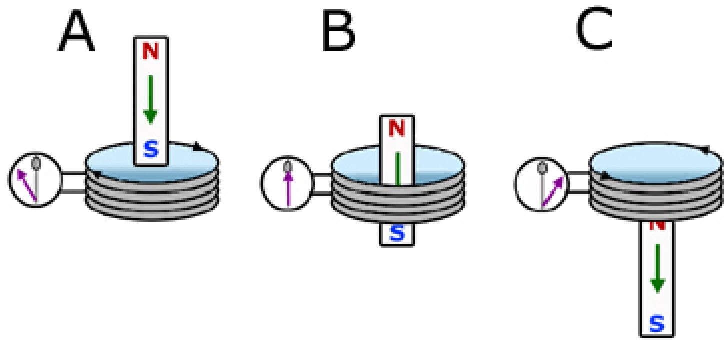
A useful application of the Hall Effect is to test whether a material is p or n-type semiconductor.

Note also how Hall Voltage V_H is inversely proportional to sample thickness ***t***. So a thin sample (a small ***t***) will give a large Hall Voltage.

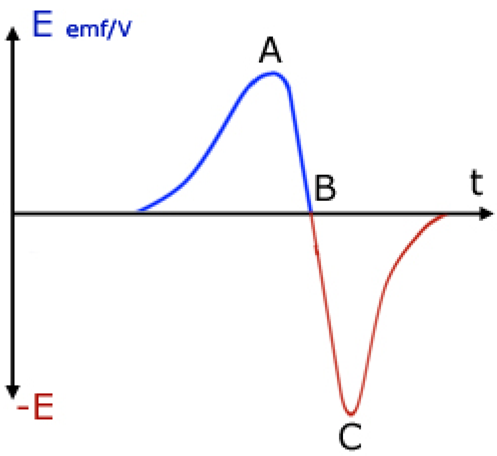
Electromagnetic Induction - Part 1

Description of the phenomenon

A rapidly changing magnetic field induces electric currents to flow in a closed circuit.



In the diagram above, a bar magnet is dropped vertically through a coil linked to a centre-zero galvanometer.



A graph of coil EMF against time shows that:

When the first pole(S) falls through the coil EMF increases to a level then decreases.

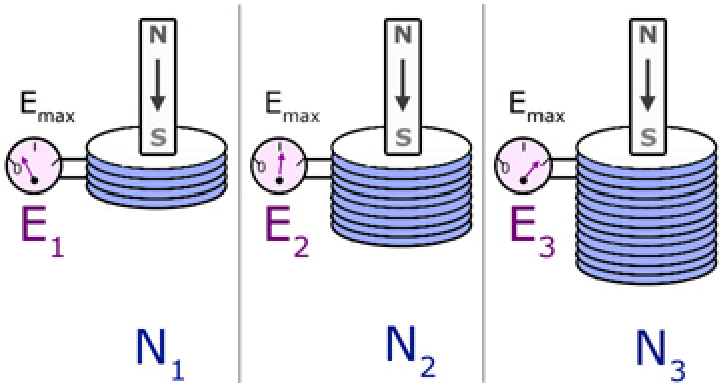
When the middle of the magnet falls through the coil, the EMF is at a minimum. No lines of force are being cut by the coil.

Maximum EMF is obtained when the second pole(N) falls through the coil. This is when the rate of cutting lines of force is highest, because the magnet is falling faster. As a result of the velocity being greater the period of high EMF is shorter.

Note that because the field direction is reversed when the poles drop through the coil, the induced current direction is also reversed. So the EMF is reversed (EMF is directly proportional to current).

Faraday's Law

Consider different sized coils when the same magnet is introduced into the body of each coil with the same velocity.



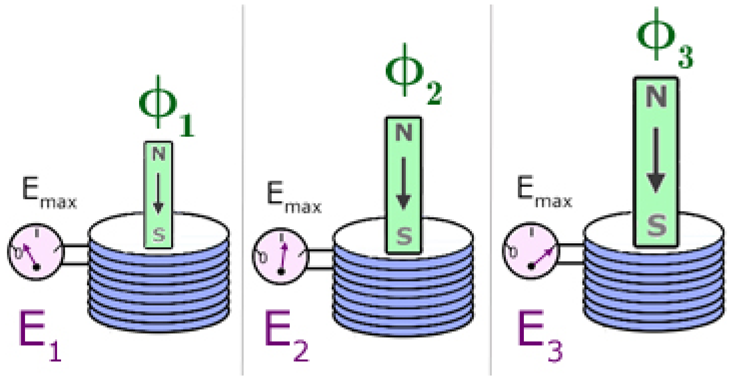
It is found that,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$$

So induced EMF ***E*** is directly proportional to number of turns ***N***,

$$E \propto N$$

Now consider just one coil and in turn introduce three magnets. The magnets are of different strengths and are introduced into the coil at the same velocity.



By measuring the maximum EMF and flux for each magnet, it is found that,

$$\frac{E_1}{\phi_1} = \frac{E_2}{\phi_2} = \frac{E_3}{\phi_3}$$

So induced EMF ***E*** is directly proportional to flux ***φ***,

$$E \propto \phi$$

Faraday's Law simply states:

The induced EMF in a closed circuit is directly proportional to the flux linkage.

Flux linkage $N\varphi$ is the product of flux φ and the number of turns N on a coil.

We have seen that,

$$E \propto N \qquad E \propto \phi$$

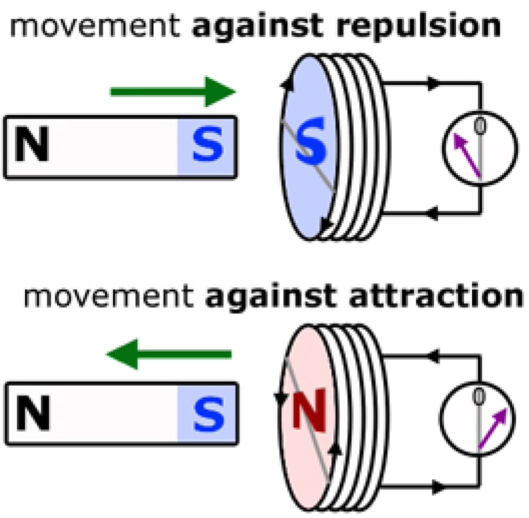
Therefore,

$$E \propto N\phi$$

Lenz's Law

The direction of the induced EMF is such that the induced current opposes the change producing it.

So when a magnetic south pole is moved towards a coil in a circuit, the face of the coil presents a south pole. The induced current is opposing the change that produced it by trying to prevent the south pole from entering the coil (by repelling it).



Similarly, when a south pole is pulled from a coil in a circuit, the face of the coil presents a north pole. The induced current is opposing the change that produced it by trying to prevent the south pole from leaving the coil (by attracting it).

Note:

1.) How the current direction is changed by the magnet direction.

2.) On each coil face, how a line drawn between the ends of arrows(in grey) makes an 'N' and an 'S' , giving the polarity of the coils.

Neumann's Equation

This combines the proportionalities in Faraday's Law with the direction of the induced current from Lenz's Law.

As a result of the consistency of units used (SI), there is no need for a constant of proportionality.

$$E = -\frac{d}{dt}(N\phi)$$

The minus sign is from Lenz's law, indicating the opposing nature of induced EMF and rate of flux linkage cutting.

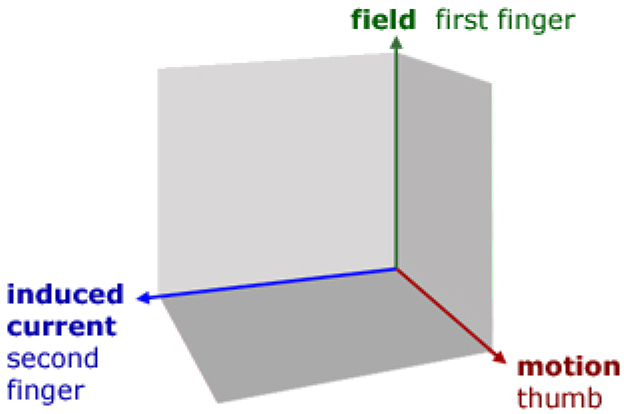
The equation can be amended to include the rate of flux cutting ***dφ/dt*** by taking the number of coils ***N*** out of the differential.

$$E = -N\frac{d\phi}{dt}$$

Flemming's Right Hand Rule

The rule describes the resulting directional motion of the induced current for a conductor moving at right angles to the field direction.

The three quantities FIELD, CURRENT AND MOTION are mutually at right angles to each other.



using the right hand, position the first finger, second finger and thumb to form the x,y,z axes. The highlighted letters within the words help you remember the three quantities

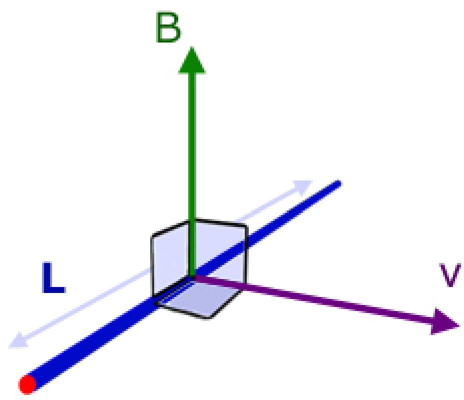
First finger - **F**ield direction

se**C**ond finger - **C**urrent direction

thu**M**b - **M**otion produced

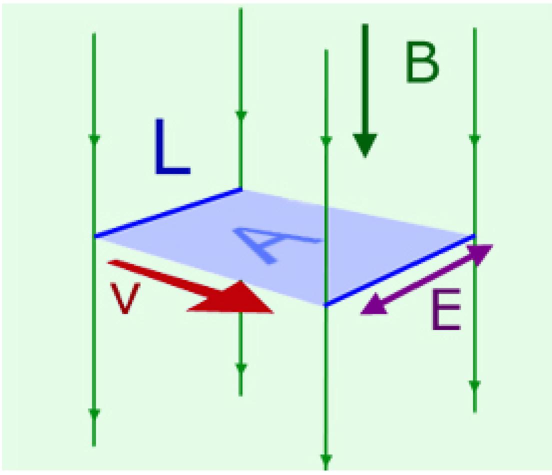
EMF induced in a metal rod

For an induced EMF ***E*** to be produced across the length ***L*** of a metal rod, the magnetic field ***B***, the velocity ***v*** and the major axis of the rod must all be mutually at right angles to each other.



The derivation of ***E = BLv*** :

Consider a metal rod of length L.



If the rod is travelling at a velocity ***v*** at right angles to its length then the area ***A*** swept out per second is given by:

$$A = Lv$$

The total flux ***φ*** threading through this area per second is the product of the area ***A*** and the flux density ***B***.

$$\phi = BA$$

Substituting for the area ***A***,

$$\phi = BLv$$

In this case, since the total flux ***φ*** refers to 1 second, we can write :

$$\phi = \frac{d\phi}{dt}$$

where ***dφ/dt*** is the rate of flux cutting.

Hence,

$$\frac{d\phi}{dt} = BLv$$

By definition, EMF (***E***) is equal to the rate of flux cutting,

$$E = \frac{d\phi}{dt}$$

Therefore,

$$E = BLv$$

Electromagnetic Induction - Part 2

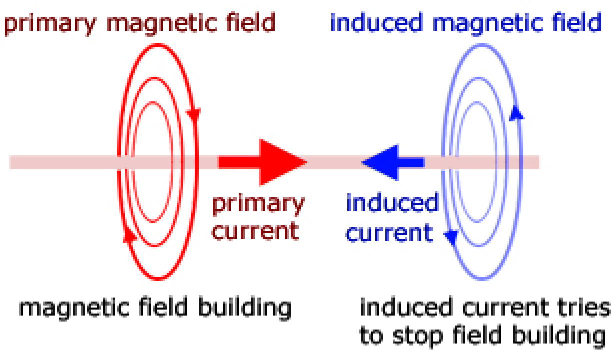
Self Induction

Induced currents only occur when a magnetic field builds or collapses.

Lenz's law tells us that the induced current is such as to oppose the change producing it.

So the induced current will oppose the primary current when the field is building. Conversely, when the field is collapsing the current will be in the opposite direction to try to prevent the collapse.

In the first instance the induced current field will be in a sense to oppose the primary current building. In the second instance when the field is collapsing, it will be in the same direction to lessen the collapse..



The induced current is produced by a 'back EMF' - an induced voltage proportional to minus the rate of primary current change.

$$\text{back EMF} \propto -(\text{rate of current change})$$

$$E = -L\frac{dI}{dt}$$

where ***L*** (the inductance) is the constant of proportionality.

Rearranging to make ***L*** the subject of the equation. By making ***E*** and ***dI/dt*** unity we define the unit of self induction, the **henry**:

$$L = -\frac{E}{\left(\frac{dI}{dt}\right)}$$

$$1 \text{ henry} = -\left(\frac{1 \text{ volt back EMF}}{1 \text{ amp/sec}}\right)$$

A coil has a self inductance of 1 henry (H) if the back EMF is 1 volt for a current change of 1 ampere/second.

By equating the EMF from Neumann's equation with self induction EMF a simplified expression linking the two can be formed.

$$-\frac{d}{dt}(N\phi) = -L\frac{dI}{dt}$$

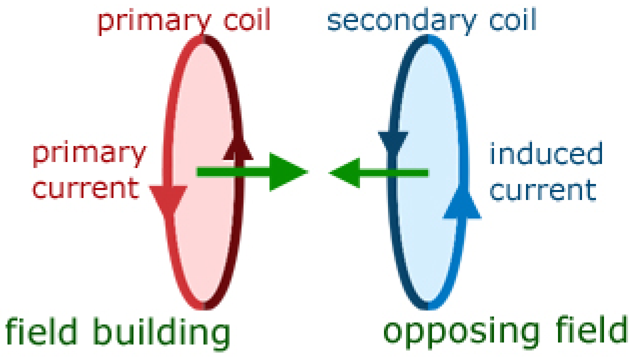
Integrating between the limits of $\boldsymbol{\varphi}$ - 0 and $\boldsymbol{I}$ - 0 ,

$$N\int_{\phi=0}^{\phi=\phi}\frac{d\phi}{dt}dt = L\int_{I=0}^{I=I}\frac{dI}{dt}dt$$

$$\underline{N\phi = LI}$$

Mutual Induction

Mutual induction concerns a pair of coils. A building current in one coil (the primary) produces a building magnetic field around it. This in turn induces a building current to flow in the other coil. The induced current flow is in such a direction as to produce a magnetic field direction to oppose the primary magnetic field. The magnetic fields are directed towards each other. So their total effect is reduced.



When the primary current direction is reversed its magnetic field direction is also reversed. This has the effect of making current in the secondary reverse direction together with its magnetic field direction. In this case the magnetic fields are in opposing directions.

As with self inductance, the back EMF is proportional to minus the rate of current change. However, in this case the back EMF is in the secondary coil, not and in the primary(as with self induction).

$$\text{back EMF} \propto -(\text{rate of current change})$$

$$E = -M\frac{dI}{dt}$$

Where ***M*** (mutual inductance) is the constant of proportionality.

There is another important link between self and mutual inductance. It can be shown, assuming 100% flux linkage that,

$$M = \sqrt{L_p L_s}$$

where L_p and L_s are respectively the self inductances of primary and secondary coils.

So the back EMF E_s in the secondary coil relates to the rate of current change dI_p/dt in the primary.

$$E_s = -M\frac{dI_p}{dt}$$

The secondary back EMF also relates to the rate of change of flux linkage in the secondary.

$$E_s = -\frac{d}{dt}(N_s\phi_s)$$

Eliminating E_s from these two expressions,

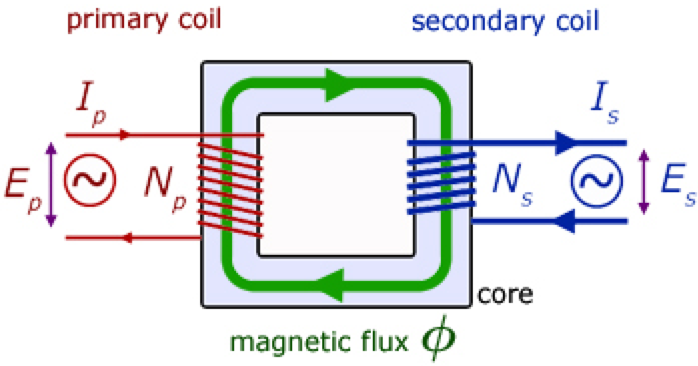
$$-\frac{d}{dt}\left(N_s\phi_s\right) = -M\frac{dI_p}{dt}$$

Integrating between the limits of φ_s - 0 and I_p - 0 ,

$$N\int_{\phi=0}^{\phi=\phi}\frac{d\phi_s}{dt}dt = M\int_{I=0}^{I=I}\frac{dI_p}{dt}dt$$

$$\underline{N_s\phi_s = MI_p}$$

Transformer action



Consider the primary coil. There are two opposing EMF's working here : the applied EMF E_p and the back EMF E_B .

If I is the current flowing in the primary and R is its resistance, then from Kirchoff's law for pd's in a circuit:

$$E_p + E_B = IR$$

Neumann's equation states that E_B is given by :

$$E_B = -\frac{d}{dt}(N_p \phi)$$

Substituting for E_B in the Kirchoff relation,

$$E_p - \frac{d}{dt}(N_p \phi) = IR$$

Assuming that the coil resistance R is so low as to be negligible, we have:

$$E_p = N_p \frac{d\phi}{dt}$$ (i

Both primary and secondary coils have the same flux passing through them. So the rate of flux change $d\varphi/dt$ will also be the same. It follows that the back EMF E_s in the secondary is given by :

$$E_s = N_s \frac{d\phi}{dt}$$ (ii

Dividing equation (ii by equation (i ,

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \qquad \text{(iii}$$

Power in a transformer

Consider a load resistance ***R*** connected to the secondary coil.
Quoting the power equation for a circuit,

$$P = IE$$

where ***P*** is power, ***I*** current and ***E*** EMF.

If we assume that there are no losses (ie that the transformer is 100% efficient) we can write :

power input = power output

If I_p and I_s are the currents flowing in the primary and secondary coils, then :

$$I_p E_p = I_s E_s$$

rearranging,

$$\frac{E_s}{E_p} = \frac{I_p}{I_s}$$

substituting for E_s / E_p from (iii the transformer equation,

$$\frac{N_s}{N_p} = \frac{I_p}{I_s}$$

Transformer efficiency

In reality the efficiency of a transformer is not 100%. However efficiency is still high, being in the range 95-99%.

$$\text{efficiency}(\%) = \frac{P_{output}}{P_{input}} \times 100$$

Ways that power is lost within a transformer :

1.) **Coil Heating** Energy is lost in the coils by resistive heating. The power loss P is given by $P = I^2R$, where R is coil resistance and I the current. This can be reduced by choosing wire thickness according to current.
Between the primary and the secondary, the coil with the smaller number of turns carries the larger current. Therefore this coil is made from thicker wire. Remember the resistance of a wire is inversely proportional to its cross-sectional area. A small cross-sectional area gives a higher resistance. A higher resistance gives a greater power loss.

2.) **Eddy Currents** Eddy Currents are unwanted induced currents formed in the body of a metal object. Much heating results from high currents induced from low EMF's.
To counteract eddy currents the core is laminated. It is constructed of very thin(approx. 1mm) sheets of *soft* iron. Each sheet is varnished and insulted from the next.

3.) **Hysteresis** The core material offers some resistance to the changing strength and direction of the magnetic field(called **hysteresis loss**). This resistance manifests itself as heat within the core.
The remedy is to make the core of specialist metal (eg permalloy, silicon steel) where hysteresis loss is minimal.

4.) **Flux** As a result of imperfections in the windings, not all the flux that passes through the primary coil passes through the secondary. This means that not all the energy is transferred between the coils.

general notes:

There are two types of transformer.
A **step-up** transformer is when $N_s > N_p$ and $E_s > E_p$.

For a **step-down** transformer, the inequality is reversed and $N_p > N_s$ and $E_p > E_s$.

The applied EMF in the primary coil must be alternating in nature. A changing magnetic field is a requirement for transformer action.